## Index

In the first part of this index, mathematical symbols and expressions are listed in their order of appearance in the book.

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