## Contents

Introduction xi  
Notation xv  

**Part 1. Matrices and Linear Dynamical Systems**

Chapter 1. Autonomous Linear Differential and Difference Equations 3  
  §1.1. Existence of Solutions 3  
  §1.2. The Real Jordan Form 6  
  §1.3. Solution Formulas 10  
  §1.4. Lyapunov Exponents 12  
  §1.5. The Discrete-Time Case: Linear Difference Equations 18  
  §1.6. Exercises 24  
  §1.7. Orientation, Notes and References 27  

Chapter 2. Linear Dynamical Systems in \( \mathbb{R}^d \) 29  
  §2.1. Continuous-Time Dynamical Systems or Flows 29  
  §2.2. Conjugacy of Linear Flows 33  
  §2.3. Linear Dynamical Systems in Discrete Time 38  
  §2.4. Exercises 43  
  §2.5. Orientation, Notes and References 43  

Chapter 3. Chain Transitivity for Dynamical Systems 47  
  §3.1. Limit Sets and Chain Transitivity 47  
  §3.2. The Chain Recurrent Set 54
Contents

§3.3. The Discrete-Time Case 59
§3.4. Exercises 63
§3.5. Orientation, Notes and References 65

Chapter 4. Linear Systems in Projective Space 67
§4.1. Linear Flows Induced in Projective Space 67
§4.2. Linear Difference Equations in Projective Space 75
§4.3. Exercises 78
§4.4. Orientation, Notes and References 78

Chapter 5. Linear Systems on Grassmannians 81
§5.1. Some Notions and Results from Multilinear Algebra 82
§5.2. Linear Systems on Grassmannians and Volume Growth 86
§5.3. Exercises 94
§5.4. Orientation, Notes and References 95

Part 2. Time-Varying Matrices and Linear Skew Product Systems

Chapter 6. Lyapunov Exponents and Linear Skew Product Systems 99
§6.1. Existence of Solutions and Continuous Dependence 100
§6.2. Lyapunov Exponents 106
§6.3. Linear Skew Product Flows 113
§6.4. The Discrete-Time Case 118
§6.5. Exercises 121
§6.6. Orientation, Notes and References 123

Chapter 7. Periodic Linear Differential and Difference Equations 127
§7.1. Floquet Theory for Linear Difference Equations 128
§7.2. Floquet Theory for Linear Differential Equations 136
§7.3. The Mathieu Equation 144
§7.4. Exercises 151
§7.5. Orientation, Notes and References 153

Chapter 8. Morse Decompositions of Dynamical Systems 155
§8.1. Morse Decompositions 155
§8.2. Attractors 159
§8.3. Morse Decompositions, Attractors, and Chain Transitivity 164
§8.4. Exercises 166
§8.5. Orientation, Notes and References 167

Chapter 9. Topological Linear Flows 169
§9.1. The Spectral Decomposition Theorem 170
§9.2. Selgrade’s Theorem 178
§9.3. The Morse Spectrum 184
§9.4. Lyapunov Exponents and the Morse Spectrum 192
§9.5. Application to Robust Linear Systems and Bilinear Control Systems 197
§9.6. Exercises 207
§9.7. Orientation, Notes and References 208

Chapter 10. Tools from Ergodic Theory 211
§10.1. Invariant Measures 211
§10.2. Birkhoff’s Ergodic Theorem 214
§10.3. Kingman’s Subadditive Ergodic Theorem 217
§10.4. Exercises 220
§10.5. Orientation, Notes and References 221

Chapter 11. Random Linear Dynamical Systems 223
§11.1. The Multiplicative Ergodic Theorem (MET) 224
§11.2. Some Background on Projections 233
§11.4. The Deterministic Multiplicative Ergodic Theorem 242
§11.5. The Furstenberg-Kesten Theorem and Proof of the MET in Discrete Time 252
§11.6. The Random Linear Oscillator 263
§11.7. Exercises 266
§11.8. Orientation, Notes and References 268

Bibliography 271

Index 279