

## Contents

Preface xi  
Acknowledgments xv  

### Part 1. Background material

<table>
<thead>
<tr>
<th>Chapter 1. Assorted notions from differential geometry</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>§1.1. Manifolds</td>
<td>3</td>
</tr>
<tr>
<td>§1.2. Tensors, indices, and the Einstein summation convention</td>
<td>9</td>
</tr>
<tr>
<td>§1.3. Differentiable maps, tangent spaces, and vector fields</td>
<td>15</td>
</tr>
<tr>
<td>§1.4. Lie groups and matrix groups</td>
<td>26</td>
</tr>
<tr>
<td>§1.5. Vector bundles and principal bundles</td>
<td>32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 2. Differential forms</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>§2.1. Introduction</td>
<td>35</td>
</tr>
<tr>
<td>§2.2. Dual spaces, the cotangent bundle, and tensor products</td>
<td>35</td>
</tr>
<tr>
<td>§2.3. 1-forms on $\mathbb{R}^n$</td>
<td>40</td>
</tr>
<tr>
<td>§2.4. $p$-forms on $\mathbb{R}^n$</td>
<td>41</td>
</tr>
<tr>
<td>§2.5. The exterior derivative</td>
<td>43</td>
</tr>
<tr>
<td>§2.6. Closed and exact forms and the Poincaré lemma</td>
<td>46</td>
</tr>
<tr>
<td>§2.7. Differential forms on manifolds</td>
<td>47</td>
</tr>
<tr>
<td>§2.8. Pullbacks</td>
<td>49</td>
</tr>
<tr>
<td>§2.9. Integration and Stokes’s theorem</td>
<td>53</td>
</tr>
<tr>
<td>§2.10. Cartan’s lemma</td>
<td>55</td>
</tr>
</tbody>
</table>
§2.11. The Lie derivative 56
§2.12. Introduction to the Cartan package for MAPLE 59

Part 2. Curves and surfaces in homogeneous spaces via the method of moving frames

Chapter 3. Homogeneous spaces 69
§3.1. Introduction 69
§3.2. Euclidean space 70
§3.3. Orthonormal frames on Euclidean space 75
§3.4. Homogeneous spaces 84
§3.5. Minkowski space 85
§3.6. Equi-affine space 92
§3.7. Projective space 96
§3.8. MAPLE computations 103

Chapter 4. Curves and surfaces in Euclidean space 107
§4.1. Introduction 107
§4.2. Equivalence of submanifolds of a homogeneous space 108
§4.3. Moving frames for curves in $\mathbb{E}^3$ 111
§4.4. Compatibility conditions and existence of submanifolds with prescribed invariants 115
§4.5. Moving frames for surfaces in $\mathbb{E}^3$ 117
§4.6. MAPLE computations 134

Chapter 5. Curves and surfaces in Minkowski space 143
§5.1. Introduction 143
§5.2. Moving frames for timelike curves in $\mathbb{M}^{1,2}$ 144
§5.3. Moving frames for timelike surfaces in $\mathbb{M}^{1,2}$ 149
§5.4. An alternate construction for timelike surfaces 161
§5.5. MAPLE computations 166

Chapter 6. Curves and surfaces in equi-affine space 171
§6.1. Introduction 171
§6.2. Moving frames for curves in $\mathbb{A}^3$ 172
§6.3. Moving frames for surfaces in $\mathbb{A}^3$ 178
§6.4. MAPLE computations 191
Chapter 7. Curves and surfaces in projective space 203

§7.1. Introduction 203
§7.2. Moving frames for curves in $\mathbb{P}^2$ 204
§7.3. Moving frames for curves in $\mathbb{P}^3$ 214
§7.4. Moving frames for surfaces in $\mathbb{P}^3$ 220
§7.5. MAPLE computations 235

Part 3. Applications of moving frames

Chapter 8. Minimal surfaces in $\mathbb{E}^3$ and $\mathbb{A}^3$ 251

§8.1. Introduction 251
§8.2. Minimal surfaces in $\mathbb{E}^3$ 251
§8.3. Minimal surfaces in $\mathbb{A}^3$ 268
§8.4. MAPLE computations 280

Chapter 9. Pseudospherical surfaces and Bäcklund’s theorem 287

§9.1. Introduction 287
§9.2. Line congruences 288
§9.3. Bäcklund’s theorem 289
§9.4. Pseudospherical surfaces and the sine-Gordon equation 293
§9.5. The Bäcklund transformation for the sine-Gordon equation 297
§9.6. MAPLE computations 303

Chapter 10. Two classical theorems 311

§10.1. Doubly ruled surfaces in $\mathbb{R}^3$ 311
§10.2. The Cauchy-Crofton formula 324
§10.3. MAPLE computations 329

Part 4. Beyond the flat case: Moving frames on Riemannian manifolds

Chapter 11. Curves and surfaces in elliptic and hyperbolic spaces 339

§11.1. Introduction 339
§11.2. The homogeneous spaces $\mathbb{S}^n$ and $\mathbb{H}^n$ 340
§11.3. A more intrinsic view of $\mathbb{S}^n$ and $\mathbb{H}^n$ 345
§11.4. Moving frames for curves in $\mathbb{S}^3$ and $\mathbb{H}^3$ 348
§11.5. Moving frames for surfaces in $\mathbb{S}^3$ and $\mathbb{H}^3$ 351
§11.6. MAPLE computations 357
Chapter 12. The nonhomogeneous case: Moving frames on Riemannian manifolds 361

§12.1. Introduction 361

§12.2. Orthonormal frames and connections on Riemannian manifolds 362

§12.3. The Levi-Civita connection 370

§12.4. The structure equations 373

§12.5. Moving frames for curves in 3-dimensional Riemannian manifolds 379

§12.6. Moving frames for surfaces in 3-dimensional Riemannian manifolds 381

§12.7. Maple computations 388

Bibliography 397

Index 403