Preface

This monograph is intended as an introduction to some elements of mathematical finance. It begins with the development of the basic ideas of hedging and pricing of European and American derivatives in the discrete (i.e., discrete time and discrete state) setting of binomial tree models. Then a general discrete finite market model is defined and the fundamental theorems of asset pricing are proved in this setting. Tools from probability such as conditional expectation, filtration, (super)martingale, equivalent martingale measure, and martingale representation are all used first in this simple discrete framework. This is intended to provide a bridge to the continuous (time and state) setting which requires the additional concepts of Brownian motion and stochastic calculus. The simplest model in the continuous setting is the Black-Scholes model. For this, pricing and hedging of European and American derivatives are developed. The book concludes with a description of the fundamental theorems of asset pricing for a continuous market model that generalizes the simple Black-Scholes model in several directions.

The modern subject of mathematical finance has undergone considerable development, both in theory and practice, since the seminal work of Black and Scholes appeared a third of a century ago. The material presented here is intended to provide students and researchers with an introduction that will enable them to go on to read more advanced texts and research papers. Examples of topics for such further study include incomplete markets, interest rate models and credit derivatives.

For reading this book, a basic knowledge of probability theory at the level of the book by Chung [10] or D. Williams [38], plus for the chapters on continuous models, an acquaintance with stochastic calculus at the level of the book by Chung and Williams [11] or Karatzas and Shreve [27], is
desirable. To assist the reader in reviewing this material, a summary of
some of the key concepts and results relating to conditional expectation,
martingales, discrete and continuous time stochastic processes, Brownian
motion and stochastic calculus is provided in the appendices. In particular,
the basic theory of continuous time martingales and stochastic calculus for
Brownian motion should be briefly reviewed before commencing Chapter 4.
Appendices C and D may be used for this purpose.

Most of the results in the main body of the book are proved in detail.
Notable exceptions are results from linear programming used in Section 3.5,
results used for pricing American contingent claims based on continuous
models in Sections 4.7 through 4.9, and several results related to the funda-
mental theorems of asset pricing for the multi-dimensional Black-Scholes
model treated in Chapter 5.

I benefited from reading treatments of various topics in other books
on mathematical finance, including those by Pliska [33], Lamberton and
Lapeyre [30], Elliott and Kopp [15], Musiela and Rutkowski [32], Bingham
and Kiesel [4], and Karatzas and Shreve [28], although the treatment
presented here does not parallel any one of them.

This monograph is based on lectures I gave in a graduate course at the
University of California, San Diego. The material in Chapters 1–3 was also
used in adapted form for part of a junior/senior-level undergraduate course
on discrete models in mathematical finance at UCSD. The students in both
courses came principally from mathematics and economics. I would like to
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