Preface

The present book, "The Classification of Finite Simple Groups: Groups of Characteristic 2 Type", completes a project of giving an outline of the proof of the Classification of the Finite Simple Groups (CFSG). The project was begun by Daniel Gorenstein in 1983 with his book [**Gor83**]—which he subtitled "Volume 1: Groups of Noncharacteristic 2 Type". Thus we regard our present discussion of groups of characteristic 2 type as "Volume 2" of that project.

The Classification of the Finite Simple Groups (CFSG) is one of the premier achievements of twentieth century mathematics. The result has a history which, in some sense, goes back to the beginnings of proto-group theory in the late eighteenth century. Many classic problems with a long history are important more for the mathematics they inspire and generate, than because of interesting consequences. This is not true of the Classification, which is an extremely useful result, making possible many modern successes of finite group theory, which have in turn been applied to solve numerous problems in many areas of mathematics.

A theorem of this beauty and consequence deserves and demands a proof accessible to any mathematician with enough background in finite group theory to read the proof. Unfortunately the proof of the Classification is very long and complicated, consisting of thousands of pages, written by hundreds of mathematicians in hundreds of articles published over a period of decades. The only way to make such a proof truly accessible is, with hindsight, to reorganize and rework the mathematics, collect it all in one place, and make the treatment self-contained, except for some carefully written and selected basic references. Such an effort is in progress in the work of Gorenstein, Lyons, and Solomon (GLS) in their series beginning with [**GLS94**], which seeks to produce a second-generation proof of the Classification.

However in the meantime, there should at least be a detailed outline of the existing proof, that gives a global picture of the mathematics involved, and explicitly lists the papers which make up the proof. Even after a second-generation proof is in place, such an outline would have great historical value, and would also provide those group theorists who seek to further simplify the proof with the opportunity to understand the approach and ideas that appear in the proof. That is the goal of this volume: to provide an overview and reader's guide to the huge literature which makes up the original proof of the Classification.

Soon after the apparent completion of the Classification in the early 1980s, Daniel Gorenstein began a project aimed at giving an outline of the original proof. He provided background in a substantial Introduction [**Gor82**], in particular discussing the partition of simple groups into groups of odd characteristic and groups of characteristic 2 type. Then in Volume 1 [**Gor83**] he described the treatment of

the groups of odd characteristic in detail. However he did not complete the rest of his project, in part because the proof for groups of characteristic 2 type remained incomplete, specifically that part of the proof treating the quasithin groups undertaken by Mason [**Mas**]. This gap was recently filled by the Aschbacher-Smith classification of the quasithin groups [**AS04b**]. Hence it is now possible to finish Gorenstein's project by outlining the proof for the groups of characteristic 2 type. We accomplish that goal here, adopting his title, and regarding the work as "Volume 2" in the series.

While we recommend that the interested reader consult Gorenstein's books, we also intend that our treatment should be sufficiently self-contained that those works will not be a prerequisite. Therefore in Chapter 1, we supply an overview of the treatment of the groups of odd characteristic, which is much briefer than Gorenstein's detailed treatment.

In fact, throughout our exposition, we will be less detailed than Gorenstein, since we believe that a briefer outline of the main steps will be more accessible and useful to most readers. On the other hand, we are careful to honor the important fundamental goal of explicitly listing those works in the literature which make up the proof that all simple groups of characteristic 2 type are known.

Mathematics, particularly the proof of a complex theorem, is hierarchical. We will list the results on groups of characteristic 2 type at the top of that hierarchy, which we refer to as "level 0" results. These are the papers containing subtheorems whose union affords the classification of the groups of characteristic 2 type. We also discuss the papers at level 1: the principal subsidiary results used in the proofs of subtheorems at level 0. We will *not* usually attempt an analysis through levels 2 and beyond; that is, as a rule we do not discuss those papers used to establish the subsidiary results, and so on, down to first principles and the level of textbooks. But our outline could be used as a starting point for such a deeper analysis of the proof.

Finally we will typically assume that the reader has some familiarity with concepts, terminology, notation, and results from elementary group theory, such as might be standard in a first year graduate algebra course. Beyond that, we will try to give more advanced definitions when they arise in our discussion. In addition we provide in Chapter A of the Appendix a review of some intermediate material on simple groups and their properties. The Index should be helpful when encountering new terminology and notation; normally the index entry given in **boldface** indicates either the definition, or the most fundamental page reference.

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