# Preface

It is in the early 2000's that persistence emerged as a new theory in the field of applied and computational topology. This happened mostly under the impulsion of two schools: the one led by H. Edelsbrunner and J. Harer at Duke University, the other led by G. Carlsson at Stanford University. After more than a decade of a steady development, the theory has now reached a somewhat stable state, and the community of researchers and practitioners gathered around it has grown in size from a handful of people to a couple hundred[1]. In other words, persistence has become a mature research topic.

The existing books and surveys on the subject [48, 114, 115, 119, 141, 245] are largely built around the topological aspects of the theory, and for particular instances such as the persistent homology of the family of sublevel sets of a Morse function on a compact manifold. While this can be useful for developing intuition, it does create bias in how the subject is understood. A recent monograph [72] tries to correct this bias by focusing almost exclusively on the algebraic aspects of the theory, and in particular on the mathematical properties of persistence modules and of their diagrams.

The goal pursued in the present book is to put the algebraic part back into context[2], to give a broad view of the theory including also its topological and algorithmic aspects, and to elaborate on its connections to quiver theory on the one hand, to data analysis on the other hand. While the subject cannot be treated with the same level of detail as in [72], the book still describes and motivates the main concepts and ideas, and provides sufficient insights into the proofs so the reader can understand the mechanisms at work.

Throughout the exposition I will be focusing on the currently most stable instance of the theory: 1-dimensional persistence. Other instances, such as multidimensional persistence or persistence indexed over general partially ordered sets, are comparatively less well understood and will be mentioned in the last part of the book as directions for future research. The background material on quiver theory provided in Chapter 1 and Appendix A should help the reader understand the challenges associated with them.

*Reading guidelines.* There are three parts in the book. The first part (Chapters 1 through 3 and Appendix A) focuses on the theoretical foundations of persistence. The second part (Chapters 4 through 7) deals with a selected set of

[1] As evidence of this, the Institute for Mathematics and its Applications at the University of Minnesota (`http://www.ima.umn.edu/`) was holding an annual thematic program on *Scientific and Engineering Applications of Algebraic Topology* in the academic year 2013-2014. Their first workshop, devoted to topological data analysis and persistence theory, gathered around 150 people on site, plus 300 simultaneous connections to the live broadcast.

[2] Let me mention a recent short survey [234] that pursues a similar goal.

applications. The third part (Chapters 8 and 9) talks about future prospects for both the theory and its applications. The document has been designed in the hope that it can provide something to everyone among our community, as well as to newcomers with potentially different backgrounds:

- Readers with a bias towards mathematical foundations and structure theorems will find the current state of knowledge about the decomposability of persistence modules in Chapter 1, and about the stability of their diagrams in Chapter 3. To those who are curious about the connections between persistence and quiver theory, I recommend reading Appendix A.
- Readers with a bias towards algorithms will find a survey of the methods used to compute persistence in Chapter 2, and a thorough treatment of the algorithmic aspects of the applications considered in Part 2.
- Practitioners in applied fields who want to learn about persistence in general will find a comprehensive yet still accessible exposition spanning all aspects of the theory, including its connections to some applications. To those I recommend the following walk through Part 1 of the document:
  a) The general introduction,
  b) Sections 1 through 3 of Chapter 1,
  c) Sections 1.1 and 2.1 of Chapter 2,
  d) Sections 1, 2.1 and 4 of Chapter 3.

  Then, they can safely read Parts 2 and 3.

For the reader's convenience, the introduction of each chapter in Parts 1 and 2 mentions the prerequisites for reading the chapter and provides references to the relevant literature. As a general rule, I would recommend reading [115] or [142] prior to this book, as these references give quite accessible introductions to the field of applied and computational topology.

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