Contents

Preface vii

Chapter 0. Preliminaries: Sets, Functions, and Induction 1
  0.1. Notation on Sets and Functions 1
  0.2. Basic Logic: Statements and Logical Connectives 4
  0.3. Sets 10
  0.4. Functions 19
  0.5. Mathematical Induction 28
  0.6. More on Sets: Axioms and Constructions 39

Chapter 1. The Real Numbers and the Completeness Property 47
  1.1. Field and Order Properties of \( \mathbb{R} \) 48
  1.2. Completeness Property of \( \mathbb{R} \) 53
  1.3. Countable and Uncountable Sets 62
  1.4. Construction of the Real Numbers 71
  1.5. The Complex Numbers 74

Chapter 2. Sequences 79
  2.1. Limits of Sequences 79
  2.2. Three Consequences of Order Completeness 92
  2.3. The Cauchy Property for Sequences 104

Chapter 3. Topology of the Real Numbers and Metric Spaces 109
  3.1. Metrics 109
  3.2. Open and Closed Sets in \( \mathbb{R} \) 115
  3.3. Open and Closed Sets in Metric Spaces 120
  3.4. Compactness in \( \mathbb{R} \) 125

v