CONTENTS

1 Overture 1
1.1 Some equations of mathematical physics 2
1.2 Linear differential operators 8
1.3 Separation of variables 12

2 Fourier Series 18
2.1 The Fourier series of a periodic function 18
2.2 A convergence theorem 31
2.3 Derivatives, integrals, and uniform convergence 38
2.4 Fourier series on intervals 43
2.5 Some applications 48
2.6 Further remarks on Fourier series 57

3 Orthogonal Sets of Functions 62
3.1 Vectors and inner products 62
3.2 Functions and inner products 68
3.3 Convergence and completeness 72
3.4 More about $L^2$ spaces; the dominated convergence theorem 81
3.5 Regular Sturm-Liouville problems 86
3.6 Singular Sturm-Liouville problems 95

4 Some Boundary Value Problems 97
4.1 Some useful techniques 98
4.2 One-dimensional heat flow 101
4.3 One-dimensional wave motion 108
4.4 The Dirichlet problem 114
4.5 Multiple Fourier series and applications 121

5 Bessel Functions 127
5.1 Solutions of Bessel's equation 128
5.2 Bessel function identities 133
5.3 Asymptotics and zeros of Bessel functions 138
5.4 Orthogonal sets of Bessel functions 143
5.5 Applications of Bessel functions 149
5.6 Variants of Bessel functions 158

6 Orthogonal Polynomials 164
6.1 Introduction 164
6.2 Legendre polynomials 166
6.3 Spherical coordinates and Legendre functions 174
6.4 Hermite polynomials 184
6.5 Laguerre polynomials 190
6.6 Other orthogonal bases 196
Contents

7 The Fourier Transform 204
7.1 Convolutions 206
7.2 The Fourier transform 213
7.3 Some applications 225
7.4 Fourier transforms and Sturm-Liouville problems 236
7.5 Multivariable convolutions and Fourier transforms 241
7.6 Transforms related to the Fourier transform 249

8 The Laplace Transform 256
8.1 The Laplace transform 256
8.2 The inversion formula 266
8.3 Applications: Ordinary differential equations 273
8.4 Applications: Partial differential equations 279
8.5 Applications: Integral equations 286
8.6 Asymptotics of Laplace transforms 292

9 Generalized Functions 303
9.1 Distributions 304
9.2 Convergence, convolution, and approximation 314
9.3 More examples: Periodic distributions and finite parts 320
9.4 Tempered distributions and Fourier transforms 330
9.5 Weak solutions of differential equations 341

10 Green’s Functions 349
10.1 Green’s functions for ordinary differential operators 350
10.2 Green’s functions for partial differential operators 358
10.3 Green’s functions and regular Sturm-Liouville problems 369
10.4 Green’s functions and singular Sturm-Liouville problems 379

Appendices
1 Some physical derivations 386
2 Summary of complex variable theory 392
3 The gamma function 399
4 Calculations in polar coordinates 404
5 The fundamental theorem of ordinary differential equations 409

Answers to the Exercises 413
References 426
Index of Symbols 429
Index 430