## Contents

Introduction ........................................... 1  
Acknowledgement ................................... 3

Lecture 1. The Classical Theory: Part I  
Beginnings of representation theory 13

Lecture 2. The Classical Theory: Part II 17

Lecture 3. Polarized Hodge Structures and Mumford-Tate Groups and Domains 31

Lecture 4. Hodge Representations and Hodge Domains 51

Lecture 5. Discrete Series and \( n \)-Cohomology 69  
Introduction ........................................ 69  
Appendix to Lecture 5: The Borel-Weil-Bott (BWB) theorem 91

Lecture 6. Geometry of Flag Domains: Part I 95  
Appendix to Lecture 6: The \( G_{\mathbb{R}} \)- and \( K_{\mathbb{C}} \)-orbit structure of \( \check{D} \) and the \( G_{\mathbb{R}} \)-orbit structure of \( \check{U} \) .................................. 120

Lecture 7. Geometry of Flag Domains: Part II 147  
Appendix to Lecture 7: The Borel-Weil-Bott theorem revisited 161

Lecture 8. Penrose Transforms in the Two Main Examples 165  
Appendix to Lecture 8: Proofs of the results on Penrose transforms for \( D \) and \( D' \) .................................. 178

Lecture 9. Automorphic Cohomology 191  
Appendix I to Lecture 9: The \( K \)-types of the TDLDS for \( SU(2,1) \) and \( Sp(4) \) .................. 209  
Appendix II to Lecture 9: Schmid’s proof of the degeneracy of the HSSS for TDLDS in the \( SU(2,1) \) and \( Sp(4) \) cases 214  
Appendix III to Lecture 9: A general result relating TDLDS and Dolbeault cohomology of Mumford-Tate domains .................. 218

Lecture 10. Miscellaneous Topics and Some Open Questions 221  
Appendix to Lecture 10: Boundary components and Carayol’s result 245

Bibliography ........................................... 299

Index .................................................. 303