INTRODUCTION

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Hervé Jacquet is one of the founders of the modern theory of automorphic representations and their associated L-functions. This volume represents a selection of his most influential papers, although a number of important works are not included because they are already available in book form. Based on several discussions with Hervé Jacquet, I have written the following introduction to try to give a flavor of the significance and prominence of his pioneering mathematical work.

The introduction of the theory of representations of adelic groups in the theory of automorphic forms had been proposed by Gelfand and his collaborators and, separately, by Godement. But it is only with the impact of the Langlands program that the proposal took full effect. Indeed, the book of Jacquet and Langlands [JL70] reexamines the work of Hecke and others, in particular the converse theorem, from the point of view of group representations. The objects of study are the automorphic representations of $GL(2)$. Such a representation may be written as a tensor product of local representations over the places of the ground field. To each local representation is attached an $L$-factor and an $\epsilon$-factor. The global $L$-function attached to the representation is the product of the local ones and the global $\epsilon$-factor (or root number) which appears in the functional function is the product of the local $\epsilon$-factors. Thus, the theory of representations reveals a more profound understanding of the functional equation of the automorphic $L$-functions than was previously seen.

There are several other, deeper motivations for the introduction of group representations in the theory of automorphic forms. For instance, the relation between modular forms for $GL(2)$ and the multiplicative group of a quaternion algebra can be formulated in a very striking way in terms of representations. This is the celebrated Jacquet-Langlands correspondence [JL70]. As a follow up, Jacquet reexamined, from the point of view of representations, the so called Rankin-Selberg convolution of two automorphic $L$-functions for $GL(2)$ [Jac72]. In the same vein, the book by Godement and Jacquet [GJ72] defined, for the first time, the standard $L$-functions attached to automorphic representations of $GL(n)$, now called Godement-Jacquet $L$-functions and proved their basic properties. Of course, in order to carry this through, it was first necessary to obtain essential properties of the irreducible representations of a $p$-adic group [Jac71].
A series of joint papers with Shalika and Piatetski-Shapiro (in various combinations) pertain to the $L$-functions of pairs, called, by analogy with the case of $GL(2)$, the Rankin Selberg $L$-functions. The Rankin Selberg $L$-functions are attached to representations of $GL(n)$ and $GL(m)$. They are the main ingredients of the converse theorem, which can be used to prove the existence of certain automorphic representations, as predicated by Langlands’ principle of functoriality ([JPSS83], [JPSS79a], [JPSS79b]). Primary examples are the construction of automorphic forms attached to cubic extensions of the ground field, the base change over a cubic extension ([JPSS79b], [JPSS81d]), and the construction of automorphic representations of $GL(3)$ obtained as the functorial image of representations of $GL(2)$ ([GJ78], joint with Gelbart). Yet another significant application is to the notion of conductor of representations of $GL(n)$ [JPSS81a]. A basic ingredient of this effort was an elaboration of the properties of Whittaker models and functions ([JPSS83], [JS85], [JS90b]), which Jacquet had made contributions to since his thesis [Jac67].

One of the most important applications of the theory of $L$-functions of pairs is contained in the two papers with Shalika ([JS81a], [JS81b]). It is a result of Langlands that an automorphic representation of $GL(n)$ is induced by automorphic cuspidal representations of various $GL(m)$. Applying results on the poles and the non-vanishing of $L$-functions of pairs, the papers proved that this decomposition is unique, a crucial result in the applications. The required non-vanishing result was first proved in [JS77] for the Godement-Jacquet $L$-functions. The method, which uses the Theory of Eisenstein series and was known to the specialists for the case of $GL(1)$, was subsequently generalized by Shahidi to the case of $L$-functions of pairs.

In the mid-eighties, Jacquet forayed into new territory and introduced a novel tool to investigate period integrals, that is, integrals of an automorphic form over a subgroup $H$ of the ambient reductive group $G$, possibly against a one dimensional character of $H$. In particular, automorphic representations which contain a vector whose period integral is non-zero are said to be distinguished by the subgroup $H$. One of the goals of this endeavor is to determine which automorphic representations are distinguished by the given subgroup. Another goal is to relate the period integrals to special values of automorphic $L$-functions. While the usual Selberg trace formula, and its generalizations by Arthur, develop an expression for the integral of a certain kernel over the diagonal, in the new investigation (relative trace formula), one integrates the kernel over appropriate subgroups. In many cases, this appears as a generalization of the Kuznetsov and Petersson formulae from the classical set-up.

In more detail, in order to study the period integral over the subgroup $H$, it is natural to integrate the kernel over the product $H \times H$. Of course, as in the standard trace formula, the integral needs to be regularized. Examples of such a construction are contained in [JL85] and [Jac86] where the integral
for the pair \((G, H)\) is compared with the integral for a pair \((G', H')\) where \(G'\) is an inner form of \(G\). In the case of [Jac86], the integral for \((G', H')\) is explicitly related to the value of an \(L\)-function and this information is transferred to the period integral over \(H\).

Still another construction is contained in a series of papers: [Jac01], [Jac03a], [Jac03b], [Jac04b], [Jac05]. There, the data is a quadratic extension \(E/F\) of number fields. The goal is to prove that if a cuspidal automorphic representation of \(GL(n, E)\) is a base change of a cuspidal representation of \(GL(n, F)\) then it is distinguished by a unitary group (the converse is easy in this case). The relative trace formula compares the (adelic) Kuznetsov-Petersson formula for \(GL(n, F)\) with a relative Kuznetsov-Petersson formula for \(GL(n, E)\), where one of the variables of the kernel is integrated over a unitary group. Once this characterization is obtained (or known for other reasons in the case of \(GL(2)\)) one can try to make more comparisons and obtain further results on period integrals. See [Jac87b] and [JC01] which gives a more detailed version of [Jac87b]. In [Jac91] a conjecture is stated, which describes representations distinguished by an orthonormal group.

Finally, [JLR93] and [JLR04] explore another idea. It is natural to try to compare a relative trace formula (with double integral over \(H \times H\)) and a standard trace formula. However, in order to make this idea works, one needs to introduce suitable weight factors.

References

References in bold appear in this volume.


Curriculum Vitae

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Academic History:
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- Member, Institute for Advanced Study, 1967 - 1969
- Associate Professor, University of Maryland, 1969 - 1970
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- Professor, Columbia University, 1974 - 2007
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Honors:
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Acknowledgment by Author

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A number of papers could have been included in this volume, the book with Langlands, the book with Godement and several joint papers with Piatetski-Shapiro and Shalika. They are available in book form from Springer-Verlag or as articles in the *Selected Works of Ilya Piatetski-Shapiro* published by the AMS.

A number of papers which appear in this volume were written in collaboration with other mathematicians. To those alive today, I want to say that the collaboration was enjoyable and fruitful. Alas, two of my most important coauthors, Piatetski-Shapiro and Shalika, passed away. I would like to dedicate this volume to their memories.