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In Chapter 12 we looked at probabilities of independent events — things that had nothing to do with one another. Here we think about probabilities in situations where we expect to see connections, such as in screening tests for diseases or DNA evidence for guilt in a criminal trial.
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