Index

Addition and extended addition rule, 17
Aigner, M., 236, 248
Algebraic structures:
  with one composition, 261
  isomorphic, 266
  over a commutative ring with identity,
    263, 264
  substructures of, 264
  with two compositions, 262, 263
Alon, N., 66
Andrews, G., 109, 116
Apostol, T., 251, 254
Arithmetic (number-theoretic) function, 97
Arrow impossibility theorem, 190

Bell numbers:
  and moments of a Poisson distribution,
    83
  and partitions of a set (resp.,
    equivalence relations), 27, 71
  exponential generating function for, 71
  recurrence for, 71
Berge, C., 159
Bertrand’s ballot problem, 138
Bijective (combinatorial) proof, 2, 8
Binomial coefficient:
  and lattice paths, 35
  as enumerator of subsets of a given
    cardinality, 31
  recurrence, 32
  table of (Pascal’s triangle), 32
Binomial inversion principle, 37
Binomial poset:
  definition, 240
  factorial function of, 240
  incidence coefficient of, 242
  reduced incidence algebra of, 243
Bogart, K., 39, 45, 195, 205
Bonferroni inequalities, 43
Burnside’s lemma, 144
Canfield, E., 203, 205
Catalan numbers:
  and Dyck words (subdiagonal lattice
    paths), 137
  and parenthesizations of a word, 136
  and triangulations of a polygon, 138
  recurrence, generating function, and
    closed form for, 136
Cauchy product, 210
Cauchy’s formula, 81
Cayley, A., 59, 66
Characteristic function of a set, 20, 39
Chi function (of an interval in a locally
  finite poset), 232
Cigler, J., 183
Circular words, 100–101
Combinatorial factorization of a
  polynomial, 4
Complete symmetric function, 92
Composition of a positive integer:
  generating functions for, 6
  pictorial representation and
    enumeration of, 2
  under restrictions on its parts, 6
weak, 5
Comtet’s theorem, 90, 93
Conjunctive normal form (of a boolean function), 157
Cycle index of a permutation, 146
Cycle numbers:
  and restricted ordered occupancy, 78
  and permutations, 79
  as connection constants, 80
  as signless Stirling numbers of the first kind, 79
  as weighted Stirling numbers, 89
  recurrence and table, 78
  recurrence and table, 79

Davis, R., 183, 186, 248
De Bruijn’s generalization of Polya’s theorem, 155, 159
Dedekind’s problem, 198
Dilworth’s antichain decomposition theorem, 195
Dilworth’s chain decomposition theorem, 196
Dilworth’s lemma, 195
Dirichlet product, 97, 210
Disjunctive normal form (of a boolean function), 157
Distribution polynomial:
  for integer partitions, 179
  for statistics on discrete structures, 175
  for the inversion statistic on integer sequences, 178
  for the inversion statistic on permutations, 176
Dobinski’s formula, 71, 81
Doubilet, P., 227, 248
Doyle, P., 40, 44

Equinumerous (equipollent, equipotent) sets, 14
Equivalence relation, 27
Erdős, P., 24, 27
Erickson, M., 65, 66
Erikson, K., 109, 116
Euler \( \phi \)-function, 98, 99
Eulerian derivative, 182
Eulerian number, 85
Exponential formula, 88

Falling factorial polynomial, 20
Fibonacci number:
  and tilings, 5
  asymptotic growth rate of, 4
  closed form for, 4
  combinatorial interpretation of, 3
  of a binomial poset, 250
  recurrence and generating function for, 3

Finite difference:
  and polynomial interpolation, 121
  antidifferences, table of, 123
  definition and basic properties of, 119
  finite difference calculus, fundamental theorem of, 122
  relation to the shift operator, 120
Finitely additive measure, 19
Ford, L., 197, 205
Formal derivative (of a formal power series), 219
Formal power series, 212
Fulkerson, D., 197, 205
Function (map, mapping, functional, transformation, operator)
  as a distribution, 15
  as a sequence or word, 15
  domain partition induced by, 22
  domain, codomain and range of, 13
  extensional and intensional conceptions of, 13
  graph of, 13
  injective, surjective and bijective, 13
  one- and two-sided inverses of, 14
  partial, 47
  weakly (resp., strictly) increasing, 44

Galois numbers of a finite vector space, ff. 181
Golden ratio, 3
Goldman, J., 182, 183
Graded poset, 239
Graham, R., 21, 27, 66, 205
Graph:
  as an irreflexive, binary relation, 57
  complete, 58
  complete bipartite, 66
  connected, 58
  edge coloring of, 62
  edge of, 57
  enumeration of isomorphism classes of, 151
  labeled and unlabeled, 58
  vertex, vertex adjacency, degree of a vertex, 57
Greatest lower bound (infimum), 204
Gross, O., 53, 54
Hall, P., 233–234
Harary, F., 62, 66, 154, 159
Harmonic numbers, 126
Harrison, M., 159
Hausdorff maximality principle, 194, 205
Incidence algebra (of a locally finite poset), 228
Indeterminate, 211
Irrelevance of alternatives (for a social welfare function), 190
Jordan–Dedekind chain condition, 239
Kaplansky, I., 40, 44
Kelley, J., 205, 259
Kirchhoff, G., 62, 66
Kitchen, J., 251, 254
Knuth, D., 21, 27, 181, 183
Kurtz, D., 203, 205

Lagrange interpolation theorem, 140
Lah numbers:
  recurrence, closed form, and table, 78
Lah numbers:
  and ordered occupancy, 77
  as connection constants, 78
  as weighted Stirling numbers, 88
  recurrence, closed form, and table, 77
Lancaster’s theorem, 93
Lattice:
  algebraic, 204
  order-theoretic, 204
  sublattice, 204
Least upper bound (supremum), 203
Legendre’s theorem, 211
Linear difference equation:
  and rational generating functions, 132
  for periodic and polynomial functions, 135
  homogeneous, with constant coefficients, 127
  in operator form, 127
  solution using its characteristic polynomial, 128
Liu, C., 130, 140
Logarithmic concavity (of a real sequence), 200
Lubell, D., 197, 205
Lucas, E.:
  and a congruence for binomial coefficients, 114
  and the problème des ménages, 40, 44
Marriage theorem, 199
Matching, 198
McCluskey, E., 158, 159
Meshalkin, L., 205
Method of linear functionals, 182
Metric:
  discrete, 212
  ultrametric, 214
Mobius function:
  of a positive integer, 98
  of an interval in a locally finite poset, 233
Mobius inversion principle:
  binomial inversion as a special case, 99
  for arithmetic functions, 98
  for bivariate functions on a locally finite poset, 235
  for univariate functions on a locally finite poset, 236
Modular binomial lattice:
  characteristic of, 247
  definition of, 247
Mulay, S., 181, 183, 259
Multinomial coefficients:
  abbreviated notation for, 51
  and distributions with prescribed occupancy numbers, 49
  and ordered partitions of a set, 50
  as enumerators of words with prescribed letter frequencies, 50
  recurrence for, 51
Multiplication rule, 20
Newton’s inequality, 203
Niven, I., 213, 223
O’Hara, K., 183
Orbit (of a permutation group), 144
Ordered direct sum decomposition of a vector space, 173
Ordered partitions of a set:
  and preferential rankings (weak orders), 52
  asymptotic formula for, 53
  recurrence and exponential generating functions for, 49–50
  infinite series for, 53
p-order of an integer, 112
Palmer, E., 147, 159

Partially ordered set (poset):
- antichain in, 194
- chain in, 194
- comparability of two elements of, 192
- covering relation between two elements of, 193
- dimension of, 194
- duality principle for, 193
- graded, 239
- length (resp., width) of, 194
- maximal chain in, 194
- minimal (resp., maximal) element of, 192, 193
- monotone boolean function on, 198
- order ideal in, 198
- smallest (resp., largest) element of, 192, 193
- subposet (induced subposet) of, 192
- weak subposet of, 192

Partition of a set:
- definition, 22
- and equivalence relations, 27
- enumeration of, by number of blocks, 69

Partition of an integer:
- as a distribution of unlabeled balls among unlabeled boxes, 101
- as a multiset of positive integers, 101
- Ferrers diagram of, 102
- generating functions for, 105–106
- pentagonal number theorem, 107
- recurrence and table, 102
- self-conjugate, 104

Pattern inventory of a permutation group, 149

Permutation group:
- as a subgroup of the symmetric group, 143
- Burnside’s lemma, 144
- orbits induced by, 144
- permutational equivalence of, 161

Permutation:
- as a bijective self-map, 15
- as a word, 15
- Cauchy’s formula, 81, 147
- cycle decomposition of, 79
- enumeration by number of cycles, 79

Pervin, W., 259

Pigeonhole principle:
- elementary form, 17

for functions, 23
for relations, 29

Polya’s first and second theorems, 146–149

Power sum:
- and Bernoulli numbers, 111
- definition, 109
- recurrences for, 109, 110

Prüfer code, 60

Principal dual order ideal (of a poset), 236

Principal order ideal (of a poset), 236

Probabilistic method (for determining bounds on Ramsey numbers), 65

Problème des rencontres:
- and the hat-check problem, 38
- solution by binomial inversion, 38

q-binomial (Gaussian) coefficient, 165
q-binomial inversion principle, 169
q-factorials of the first and second kinds, 163–164
q-integer, 163
q-multinomial coefficients of the first and second kind, 172
q-Vandermonde identity, 171

Quasi-order (preorder):
- as a partially ordered partition, 207
- as a reflexive, transitive relation, 188
- connection with topologies, 189

Quine, W., 158, 159

Ramsey, F., 62, 66

Rank function (of a poset), 203

Rational generating functions
- (fundamental theorem of), 132

Reciprocal polynomial, 4

Relation:
- covering, in a partially ordered set, 193
- domain and range of, 24
- dual, complement, symmetric and asymmetric part of, 26
- graph of, 24
- Intensional and extensional conception of, 24
- matrix representation of, 26
- pigeonhole principle for, 30
- symmetric complement of, 191
- types of (reflexive, symmetric, asymmetric, antisymmetric, transitive, complete, negatively transitive), 25

Restricted growth function, 72
Rising factorial polynomial, 21
Rota, G.-C., 19, 27, 81, 182, 183, 227, 248, 249
Rothschild, B., 62, 66
Schoenfield, J., 215, 223
Schur’s lemma, 65
Semigroup algebra, 209
Sen, A., 190, 205
Shattuck, M., 181, 183
Sieve formula (principle of inclusion and exclusion):
abstract form (inversion formula for set functions), 41
basic form, 19
complementary form, 19
noninductive proof of, 39
Snake oil method, 123
Social welfare function, 190
Spanning subset (of a vector space), 183
Spanning tree (of a connected graph), 62
Spencer, J., 64, 66, 81
Spener poset, 203
Spener’s theorem, 197
Spiegel, M., 122, 140
Stanley, R., 11, 27, 131, 227, 248, 249
Stirling numbers of the first kind:
and elementary symmetric functions, 75
as connection constants, 75
recurrence and table, 76
signless, 79
Stirling numbers of the second kind:
and restricted growth functions, 72
and set partitions with prescribed number of blocks, 69
as connection constants, 74
exponential generating function for, 70
recurrence for, 70
table of (Stirling’s triangle), 70
Stone, H., 159
Strict order, 191
Strong convergence (of a complex sequence), 213
Strong logarithmic concavity (of a real sequence), 200
Strong pointwise convergence:
of a sequence in $C^N$, 213
of an infinite series in $C^N$, 213
of sequences and series in $C^{\text{int}(P)}$, 229
Summability:
of a complex sequence, 216, 251
of a double sequence in $C^N$, 217
of a sequence in $C^{\text{int}(P)}$, 230
of a sequence in $C^N$, 216
Sylvester, J., 104, 105
System of distinct representatives (SDR), 198–199
Székeres, A., 24, 27
Szpilrajn’s theorem, 194, 205
Top-down summation, 7
Topology:
base of, 255
definition of, 255
discrete, 255
indiscrete, 255
metric, 256
of pointwise convergence, 257
product, 257
separability axioms for, 257
Total (linear) order, 187
Touchard, J., 39, 44
Tournament, 29
Tree:
as connected graph with no cycles, 59
Cayley’s formula for the number of, 59
Kirchhoff’s matrix tree theorem, 62
Prüfer code of, 60, 61
Trotter, W., 195, 205
Twenty-fold way, 77, 78
Unimodal sequence of real numbers, 167, 200
Vacuous implication, principle of, 12
Vandermonde, A.-T.:
binomial coefficient identity, 33
determinant factorization theorem, 129
Velleman, D., 14, 27, 194, 205
Wade, W., 36, 251, 254
Wagner, C., 109, 117, 180, 183
Warner, S., 210, 223, 261, 266
Weak order:
as a reflexive, transitive, complete relation, 188
connection with ordered partitions, 189
Weak Pareto property (of a social welfare function), 190
Weight function on the positive integers:
definition, 87
weighted Stirling and Bell numbers, 88
Well-ordering, 187
West, D., 59, 66
Wilf, H., 44, 46, 123, 140
Worpitsky’s identity, 85
Zeilberger, D., 183
Zeta function:
   of a positive integer, 98
   of an interval in a locally finite poset, 231