Introduction

This book is an introduction to Euclidean plane geometry with axioms based on rigid motions. The initial spark for thinking about this topic was my reaction to reading the Common Core State Standards for Mathematics (CCSSM), especially this statement:

Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.¹

For almost all geometry textbooks, this statement would not make sense. Even in textbooks with a focus on geometrical transformations, such as Barker and Howe [1], the Side-Angle-Side criterion for triangle congruence (SAS) is taken as an axiom and is then used to prove the existence and properties of rigid motions.

In contrast, a basic assumption of the CCSSM for plane geometry is the existence of rigid motions, transformations that preserve both distance and angle measure. The triangle congruence criteria are then proved as theorems. The rationale for this approach is that such a path into geometry is intuitive, easily modeled informally, and arrives at interesting theorems sooner than by other routes. Moreover, this approach gives students the valuable tools of geometrical transformations at an early stage along with all the traditional geometry tools. Hung-Hsi Wu [19] expounds this pedagogical point of view eloquently and in detail.

So what would the rigorous mathematics of such an approach look like? What would the axioms be? How would the choice and flavor of topics be changed? And what would be the implications for teachers and students?

I was presented a rare opportunity through my association with the IAS/Park City Mathematics Institute (PCMI)² to ponder and respond to these questions. We in the PCMI program for teachers decided to develop a short online geometry course for teachers reflecting the Common Core approach to geometry. Working with my colleague and co-leader Gabriel Rosenberg, over two summers we collaborated with teachers developing ideas for such a course. Then, with Gabe as instructor, we offered the

¹CCSSM [14], standard CCSS.MATH.CONTENT.HSG.CO.B.8
²PCMI is a program of the Institute for Advanced Study in Princeton, New Jersey.
course several times in a videoconferencing format. The five lessons for that course provided an initial outline for this book.

**Transformations and Secondary Geometry**

For about a century, mathematicians and educators have recommended a greater role for transformations in the geometry curriculum, but for the most part, only small steps have taken place.

As noted above, a difficulty has been that, with the most commonly used sets of axioms, the geometry required to define basic transformations, such as reflections, rotations, and translations, is developed fairly late in the course, when many opportunities for using these tools have already passed. This means that transformations appear more as an advanced topic or an enrichment topic than as a central part of geometry.

The path urged by the Common Core makes these tools available at the very beginning in an accessible way. The key to this approach is to assume the existence of transformations that preserve both distance and angle, not just distance. This will be explained further, beginning in Chapter 1.

Until recently, a second difficulty with introducing transformations into school geometry was that they seemed less visual and more abstract than figures. It is not clear how to draw a transformation. Pictures can show polygons more vividly than functions. This problem has been alleviated in recent years by the availability of dynamic geometry software that provides movement and interaction with transformations in a way that was impossible on paper.

With such a dramatic change being proposed in the flow of the geometry curriculum, there’s a concern that teachers are being asked to teach in a new way while textbooks are still organized along an earlier model. There have been only a few resources that reflect the Common Core approach in useful detail.

While this is not a high school textbook, I hope that anyone interested will see in the early chapters of this book a way to arrive at familiar territory of congruent triangles while also taking advantage of new tools. This is then followed up by topics such as parallel lines and similarity, intertwined with half-turns, translations, and dilations. I also hope that this book will be helpful to college and university instructors teaching geometry, especially when their students are future teachers who will want and need to understand the Common Core approach.

Most of all, I hope that fellow lovers of geometry will find this an interesting path into their favorite subject.

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