Index

Arbitrage, 225
Arcsine law, 63, 194
Asian options, 245

Bachelier model, 242
Bachelier's formula, 89, 206
Backward equation, 184, 186
Bernoulli random variable, 7
Bessel process, 167
Binomial random variable, 7
Black-Scholes model, 221, 227
Black-Scholes PDE, 227, 228, 240
Bond (zero-coupon), 248
Box-Mueller method, 27
Brownian bridge, 37, 79
Brownian motion, 36, 51
Brownian motion in $\mathbb{R}^d$, 135
Brownian motion with drift, 37

Call option, 221
Cameron-Martin theorem, 202, 203, 209
Cauchy random variable, 12
Central limit theorem, 18
Chebyshev's inequality, 17
Chernoff's inequality, 17
Cholesky decomposition, 31
Completeness, 110
Conditional expectation, 70, 72, 75, 77
Convergence almost surely, 58
Convergence in $L^2$, 44
Convergence in probability, 57
Covariance, 25
Covariance matrix, 25
Cox-Ingersoll-Ross (CIR) model, 168, 247

Cumulative distribution function (CDF), 6
Derivatives, 221
Diffusion, 154
Discounting process, 220
Distribution, 5, 23
Dominated convergence theorem, 86
Doob-Meyer decomposition, 83, 125
Dynkin's formula, 147

Equivalent probabilities, 202
Euler-Maruyama scheme, 164
Exotic options, 245
Expectation, 11
Exponential random variable, 7

Feynman-Kac formula, 192
Filtration, 80
Finite-dimensional distributions, 35
Forward contract, 222
Forward equation, 190
Fractional Brownian motion, 38
Fundamental theorems of asset pricing, 243

Gambler's ruin, 87, 123, 208
Gaussian process, 35
Gaussian vector, 28
Generator, 185, 188
Geometric Brownian motion, 81
Giranov's theorem, 211
Greeks, 232

Harmonic extension, 147
Harmonic function, 143
Heat equation, 183
Heavy-tailed random variables, 13
Hedging (Delta and Gamma), 234
Hessian matrix, 138
Heston model, 251
Hull-White model, 247

IID decomposition, 30
IID random variables, 25
Implied volatility, 235
Indicator function, 5
Inner product, 41
Interest rate, 246
Itô integral, 104
Itô process, 153
Itô's formula, 114, 115, 121, 139, 143, 158, 162, 163
Itô's isometry, 109

Jensen's inequality, 79
Law of large numbers, 17, 44, 57, 58
Lévy's characterization theorem, 170
Lookback options, 245

Marginal distribution, 25
Market model, 220
Markov process, 84, 178
Markov property (strong), 181
Markov's inequality, 16
Martingale, 80
Martingale representation theorem, 170
Martingale transform, 100
Maximal inequalities, 96
Müller scheme, 164
Moment generating function (MGF), 15
Monotone convergence theorem, 59
Monte-Carlo pricing, 241

Novikov's condition, 163
Option portfolio, 222
Optional stopping, 86
Ornstein-Uhlenbeck process, 38, 54, 159

Poisson process, 61, 82
Poisson random variable, 7
Pricing formula, 239
Probability density function (PDF), 9, 24
Probability space, 4
Projection, 42
Put option, 221
Put-call parity, 222, 226, 239
Random variable, 4
Random vector, 23
Random walk, 82
Recurrence, 144
Reflection principle, 90
Replicating portfolio, 225
Riemann-Stieltjes integral, 99
Risk-neutral pricing, 236
Risk-neutral probability, 237

Sigma-field, 4, 74
Smoluchowski's equation, 191
Space $L^2$, 40
Spaces $L^p$, 79
Stochastic differential equation (SDE), 154, 163, 165
Stochastic process, 34
Stopping time, 85
Submartingale, 80
Supermartingale, 80
Tanaka's formula, 125
Transience, 144
Transition probability density, 178
Uncorrelated random variables, 25
Uniform random variable, 7
Valuation equation, 227
Variation, 55
Variation (quadratic), 56, 156
Vasicek model, 247
Wiener integral, 112