Preface

The theory of numbers is the last great uncivilized continent of mathematics. It is split up into innumerable countries, fertile enough in themselves, but all the more or less indifferent to one another's welfare and without a vestige of a central, intelligent government. If any young Alexander is weeping for a new world to conquer, it lies before him.

Eric Temple Bell

The elementary theory of numbers should be one of the very best subjects for early mathematical instruction. It demands very little previous knowledge; its subject matter is tangible and familiar; the processes of reasoning which it employs are simple, general, and few; and it is unique among the mathematical sciences in its appeal to natural human curiosity. A month's intelligent instruction in the theory of numbers ought to be twice as instructive, twice as useful, and at least ten times as entertaining as the same amount of "calculus for engineers."

Godfrey Harold Hardy

Number theorists are like lotus-eaters—having once tasted of this food they can never give it up.

Leopold Kronecker

Some time ago I was looking at several textbooks for the undergraduate number theory course. I was struck by how few illustrations were included in many of those textbooks. A number—specifically a positive integer—can represent many things: the cardinality of a set; the length of a line segment; or the area of a plane region. Such representations naturally lead to a variety of visual arguments for topics in elementary number theory. Since the number theory course usually begins with properties of the positive integers, the texts should have more pictures. That observation became the motivation for this book.

Work on this book began when I was invited to give a talk at the MAA’s MathFest in Albuquerque in August 2005, in a session entitled “Gems of Number Theory” organized by Arthur Benjamin and Ezra
Brown. The title of that talk was “Some visual gems from elementary number theory.” Later a version of the talk appeared as an article in the February 2008 issue of *Math Horizons*. This article was subsequently included in the 2009 MAA book *Biscuits of Number Theory*, edited by Benjamin and Brown.

*Nuggets of Number Theory: A Visual Approach* is not a textbook. Although it is designed to be used by the instructor of an undergraduate number theory course as a supplement to a standard textbook, it will be of interest to anyone who loves number theory. Certain chapters in the book may be of interest to those who teach discrete mathematics, abstract algebra, and teacher preparation courses. The book can also be used as a resource for group projects or extra-credit assignments.

A *nugget* is a lump of precious metal or, more generally, something of significance or of great value. The nuggets in this book are topics in number theory for which I believe a visual approach is appropriate and beneficial, with a chapter devoted to each one. Chapter 1 is devoted to figurate numbers—numbers that can be represented by objects such as pebbles arranged in geometrical patterns—that were studied by the early Greeks. Chapter 2 deals with the important concept of congruence, and includes visual demonstrations of Fermat’s little theorem and Wilson’s theorem. Visual approaches to the solutions to linear Diophantine equations and Pell equations are the subject of Chapter 3. Pythagorean triples—solutions to the Diophantine equation $a^2 + b^2 = c^2$—are represented naturally by integer-sided right triangles and explored in Chapter 4. Certain irrational numbers and their rational approximations, including ones based on continued fractions, appear in Chapter 5. Many identities for the Fibonacci and Lucas numbers are presented visually in Chapter 6, and similarly for properties of perfect numbers in Chapter 7. Each chapter includes a set of exercises, with solutions to all the exercises following the final chapter. The book concludes with references and a complete index.

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Roger B. Nelsen
Lewis & Clark College
Portland, Oregon