Contents

Chapter 1. Introduction 1
  1.1. Outline of the proof 4
  1.2. Outline of manuscript 8
  1.3. Other approaches 8
  1.4. Acknowledgements 10

Part 1. GEOMETRIC AND ANALYTIC RESULTS FOR RICCI FLOW WITH SURGERY 11

Chapter 2. Ricci flow with surgery 13
  2.1. The standard solution and surgery 14
  2.2. Main existence theorem 15
  2.3. Review of notation and definitions 17
  2.4. Geometric limits of Ricci flows 21

Chapter 3. Limits as $t \to \infty$ 25
  3.1. Preliminary estimates on curvature and volume 25
  3.2. Three propositions 29
  3.3. The hyperbolic pieces 30
  3.4. Locally volume collapsed part of the $(M_t, g(t))$ 39

Chapter 4. Local results valid for large time 41
  4.1. First local result 41
  4.2. Second local result 66
  4.3. A corollary 77

Chapter 5. Proofs of the three propositions 79
  5.1. Proof of Proposition 3.2.4 79
  5.2. Proof of Proposition 3.2.3 85
  5.3. Proof of Proposition 3.2.1 86

Part 2. LOCALLY VOLUME COLLAPSED 3-MANIFOLDS 93

Chapter 6. Introduction to Part II 95
  6.1. Seifert fibered manifolds and graph manifolds 95
  6.2. The statement 97
  6.3. Stronger results 99
# CONTENTS

Chapter 7.  The collapsing theorem  
  7.1.  First remarks  
  7.2.  The collapsing theorem  
  7.3.  Proof that Theorem 7.2.1 implies Theorem 6.2.1  

Chapter 8.  Overview of the rest of the argument  

Chapter 9.  Basics of Gromov-Hausdorff convergence  
  9.1.  Limits of compact metric spaces  
  9.2.  Limits of complete metric spaces  
  9.3.  Manifolds with curvature bounded below  

Chapter 10.  Basics of Alexandrov spaces  
  10.1.  Properties of comparison angles  
  10.2.  Alexandrov spaces of curvature $\geq 0$  
  10.3.  Strainers  
  10.4.  Alexandrov balls  
  10.5.  The tangent cone  
  10.6.  Consequences of the existence of tangent cones  
  10.7.  Directional derivatives  
  10.8.  Blow-up results  
  10.9.  Gromov-Hausdorff limits of balls in the $M_n$  

Chapter 11.  2-dimensional Alexandrov spaces  
  11.1.  Basics  
  11.2.  The interior  
  11.3.  The boundary  
  11.4.  The covering  
  11.5.  Transition between the 2- and 1-dimensional parts  

Chapter 12.  3-dimensional analogues  
  12.1.  Regions of $M$ near generic 2-dimensional points  
  12.2.  The global $S^1$-fibration  
  12.3.  Balls centered at points of $\partial M_n$  
  12.4.  The interior cone points  
  12.5.  Near almost flat boundary points  
  12.6.  Boundary points of angle $\leq \pi - \delta$  
  12.7.  Balls near open intervals  
  12.8.  Determination of the constants  

Chapter 13.  The global result  
  13.1.  Part of $M$ close to intervals  
  13.2.  Part of $M$ near 2-dimensional Alexandrov spaces  
  13.3.  Fixing the 3-balls and attaching solid cylinders  
  13.4.  $\epsilon$-chains
13.5. The Seifert fibration containing $W_2 \setminus \cup_i U(C'_i)$ 230
13.6. Deforming the boundary of $W_2$ 232
13.7. Removing solid tori and solid cylinders from $W_2$ 239
13.8. Completion of the proof 241

Part 3. THE EQUIVARIANT CASE 243

Chapter 14. The equivariant case 245
14.1. The statement 245
14.2. Preliminary results on compact group actions 246
14.3. Actions on canonical neighborhoods 251
14.4. Equivariant Ricci flow with surgery 261
14.5. Proof of Theorem 14.1.4. 271

Bibliography 281
Glossary of symbols 283
Index 289