## Contents

Preface v

Chapter 1. Geometric Constructions of the Irreducible Representations of \( GL_n \)

JOEL KAMNITZER 1

Introduction 1
1.1. Representation theory of \( GL_n \) 2
1.2. Borel-Weil theory 6
1.3. Ginzburg construction 9
1.4. Geometric Satake correspondence 12
1.5. Geometric skew Howe duality 15

Chapter 2. Introduction to Crystal Bases

SEOK-JIN KANG 19

Introduction 19
2.1. Lie algebras 20
2.2. Kac-Moody algebras 23
2.3. Quantum groups 26
2.4. Crystal bases 28
2.5. Abstract crystals 31
2.6. Perfect crystals 33
2.7. Combinatorics of Young walls 38

Chapter 3. Geometric Realizations of Crystals

ALISTAIR SAVAGE 45

Introduction 45
3.1. Motivating examples 46
3.2. Quivers 50
3.3. The Lusztig quiver variety 55
3.4. The lagrangian Nakajima quiver variety 58
3.5. Connections to combinatorial realizations of crystal graphs 63

Chapter 4. Nilpotent Orbits and Finite \( W \)-Algebras

WEIQIANG WANG 71

Introduction 71
4.1. Nilpotent orbits, Dynkin, and good \( Z \)-gradings 74
4.2. Definitions of \( W \)-algebras 78
4.3. Quantization of the Slodowy slices 82
4.4. An equivalence of categories 86
4.5. Good $\mathbb{Z}$-gradings in type $A$ & 89  
4.6. $W$-algebras and independence of good gradings & 92  
4.7. Higher level Schur duality & 96  
4.8. $W$-(super)algebras in positive characteristic & 99  
4.9. Further work and open problems & 103  

Chapter 5. Extended Affine Lie Algebras – An Introduction to Their Structure Theory  

ERHARD NEHER  

Introduction & 107  
5.1. Affine Lie algebras and some generalizations & 109  
5.2. Extended affine Lie algebras: Definition and first examples & 120  
5.3. The structure of the roots of an EALA & 129  
5.4. The core and centreless core of an EALA & 144  
5.5. The construction of all EALAs & 156  

Chapter 6. Representations of Affine and Toroidal Lie Algebras  

VYJAYANTHI CHARI  

Introduction & 169  
6.1. Simple Lie algebras & 170  
6.2. Affine Lie algebras & 176  
6.3. Affine Lie algebras integrable representations and integral forms & 180  
6.4. Finite-dimensional modules for loop algebras and their generalizations & 183  
6.5. Weyl modules, restricted Kirillov–Reshetikhin and beyond & 187  
6.6. Koszul algebras, quivers, and highest weight categories & 191  

Bibliography & 199  

Index & 209