Preface

The theory of nonlinear elliptic partial differential equations of the second order has flourished in the past half-century. The pioneering work of de Giorgi in 1957 opened the door to the study of general quasilinear elliptic differential equations. Since then, the nonlinear elliptic differential equation has become a diverse subject and has found applications in science and engineering. In mathematics, the development of elliptic differential equations has influenced the development of the Riemannian geometry and complex geometry. Meanwhile, the study of elliptic differential equations in a geometric setting has provided interesting new questions with fresh insights to old problems.

This book is written for those who have completed their study of the linear elliptic differential equations and intend to explore the fascinating field of nonlinear elliptic differential equations. It covers two classes of nonlinear elliptic differential equations, quasilinear and fully nonlinear, and focuses on two important nonlinear elliptic differential equations closely related to geometry, the mean curvature equation and the Monge-Ampère equation.

This book presents a detailed discussion of the Dirichlet problems for quasilinear and fully nonlinear elliptic differential equations of the second order: quasilinear uniformly elliptic equations in arbitrary domains, mean curvature equations in domains with nonnegative boundary mean curvature, fully nonlinear uniformly elliptic equations in arbitrary domains, and Monge-Ampère equations in uniformly convex domains. Global solutions of these equations are also characterized. The choice of topics is influenced by my personal taste. Some topics may be viewed by others as too advanced for a graduate textbook. Among those topics are the curvature estimates for minimal surface equations, the complex Monge-Ampère equation, and the
generalized solutions of the (real) Monge-Ampère equations. Inclusion of these topics reflects their importance and their connections to many of the most active current research areas.

There is an inevitable overlap with the successful monograph by Gilbarg and Trudinger. This book, designed as a textbook, is more focused on basic materials and techniques. Many results in this book are presented in special forms. For example, the quasilinear and fully nonlinear uniformly elliptic differential equations studied in this book are not in their most general form. The study of these equations serves as a prerequisite to the study of the mean curvature equation and the Monge-Ampère equation, respectively. More notably, our discussion of the Monge-Ampère equations is confined to the pure Monge-Ampère equations, instead of the Monge-Ampère type equations.

This book is based on one-semester courses I taught at Peking University in the spring of 2011 and at the University of Notre Dame in the fall of 2011. Part of it was presented in the Special Lecture Series at Peking University in the summer of 2007, in the Summer School in Mathematics at the University of Science and Technology of China in the summer of 2008, and in a graduate course at Beijing International Center of Mathematical Research in the spring of 2010.

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