This book is an introduction to the theory of quiver representations and quiver varieties. It is based on a course given by the author at Stony Brook University. It begins with basic definitions and ends with Nakajima’s work on quiver varieties and the geometric realization of Kac–Moody Lie algebras.

The book aims to be a readable introduction rather than a monograph. Thus, while the first chapters of the book are mostly self-contained, in the second half of the book some of the more technical proofs are omitted; we only give the statements and some ideas of the proofs, referring the reader to the original papers for details.

We tried to make this exposition accessible to graduate students, requiring only a basic knowledge of algebraic geometry, differential geometry, and the theory of Lie groups and Lie algebras. Some sections use the language of derived categories; however, we tried to reduce their use to a minimum.

The material presented in the book is taken from a number of papers and books (some small parts are new). We provide references to the original works; however, we made no attempt to discuss the history of the work. In many cases the references given are the most convenient or easy to read sources, rather than the papers in which the result was first introduced. In particular, we heavily used Crawley-Boevey’s lectures [CB1992], Ginzburg’s notes [Gin2012], and Nakajima’s book [Nak1999].

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