# Contents

List of Notation xiii  
Preface xvii  
Introduction 1  
  A brief history 1  
  What’s in this book 3  
  Some things not in this book 5  
  Some possible paths through this book 6  

Part 1. Background  
Chapter 1. Basic Dynamics on $\mathbb{P}^1(K)$ 11  
  §1.1. Elementary discrete dynamics 11  
  §1.2. Morphisms and coordinate changes 13  
  §1.3. Degrees, multiplicities, and multipliers 15  
  §1.4. Critical points and exceptional sets 19  
  §1.5. Dynamics in degree less than 2 22  
  §1.6. An overview of complex dynamics 23  
Exercises for Chapter 1 26  

Chapter 2. Some Background on Non-Archimedean Fields 33  
  §2.1. Absolute values 33  
  §2.2. Disks 38  
Exercises for Chapter 2 42
Chapter 3. Power Series and Laurent Series 47
  §3.1. Convergence of power series 47
  §3.2. Power series rings 49
  §3.3. Newton polygons 52
  §3.4. Images of disks under power series 56
  §3.5. $\mathbb{P}^1(C_v)$-disks and affinoids 59
  §3.6. Laurent series on open annuli 61
  §3.7. Images of annuli 62
Exercises for Chapter 3 64

Part 2. Elementary Non-Archimedean Dynamics

Chapter 4. Fundamentals of Non-Archimedean Dynamics 71
  §4.1. Classifying periodic points 71
  §4.2. Local conjugacies at fixed points (optional) 73
  §4.3. Good reduction 75
  §4.4. Lattès maps 80
  §4.5. Dynamics on disks 85
Exercises for Chapter 4 90

Chapter 5. Fatou and Julia Sets 97
  §5.1. The spherical metric 97
  §5.2. Fatou and Julia sets 99
  §5.3. Further properties of Fatou and Julia sets 105
  §5.4. Examples of non-archimedean Fatou and Julia sets 110
Exercises for Chapter 5 116

Part 3. The Berkovich Line

Chapter 6. The Berkovich Projective Line 121
  §6.1. Seminorms as Berkovich points 122
  §6.2. Disks in the Berkovich affine line 125
  §6.3. Berkovich’s classification 128
  §6.4. The Berkovich projective line 129
  §6.5. Disks and affinoids in $\mathbb{P}^1_{an}$ 133
  §6.6. Paths and path-connectedness 138
  §6.7. Directions at Berkovich points 143
  §6.8. The hyperbolic metric 144
Contents

Exercises for Chapter 6 146

Chapter 7. Rational Functions and Berkovich Space 153
§7.1. The action of rational functions 153
§7.2. Images of points of Types II and III 156
§7.3. Local degrees in directions 159
§7.4. Local degrees at Berkovich points 162
§7.5. Computing local degrees 167
§7.6. The injectivity and ramification loci 170
Exercises for Chapter 7 173

Part 4. Dynamics on the Berkovich Line

Chapter 8. Introduction to Dynamics on Berkovich Space 183
§8.1. Berkovich Fatou and Julia sets 183
§8.2. Classifying Berkovich periodic points 185
§8.3. Good reduction in Berkovich space 192
§8.4. More basic properties of Berkovich Julia sets 194
§8.5. Examples 195
Exercises for Chapter 8 198

Chapter 9. Classifying Berkovich Fatou Components 203
§9.1. Berkovich Fatou components 203
§9.2. Attracting components 206
§9.3. Indifferent components 211
§9.4. Rivera-Letelier’s classification 220
Exercises for Chapter 9 226

Chapter 10. Further Results on Periodic Components 231
§10.1. Periodic points in the indifference domain 231
§10.2. Indifferent components in mixed characteristic 235
§10.3. Counting cycles of Fatou components 240
§10.4. Infinitely many periodic components 245
Exercises for Chapter 10 247

Chapter 11. Wandering Domains 253
§11.1. Wandering domains are eventually disks 253
§11.2. An expansion theorem 256
§11.3. No wandering domains for p-adic fields: A theorem 267
Contents

§15.4. Proving compactness 396
§15.5. Proving path-connectedness 399
§15.6. Other Berkovich space proofs 405
Exercises for Chapter 15 409

Chapter 16. Proofs of Results on Berkovich Maps 411
§16.1. Basic results on Berkovich maps 411
§16.2. Proofs on local degrees 416
§16.3. Proving Rivera-Letelier’s reduction theorem 423
Exercises for Chapter 16 426

Appendices

Appendix A. Fatou Components without Berkovich Space 429
§A.1. Analytic components and $D$-components 429
§A.2. Examples of non-archimedean Fatou components 431
§A.3. Open analytic components 433
Exercises for Appendix A 435

Appendix B. Other Constructions of Berkovich Spaces 437
§B.1. Berkovich disks via power series 437
§B.2. Seminorms in homogeneous coordinates 440
§B.3. Berkovich morphisms via homogeneous coordinates 443
§B.4. Rivera-Letelier’s construction of $\mathbb{P}^1_{\text{an}}$ 445
Exercises for Appendix B 447

Bibliography 449

Index 457