Contents

Preface ix

Chapter I. Normal forms and desingularization 1
1. Analytic differential equations in the complex domain 1
2. Holomorphic foliations and their singularities 29
3. Formal flows and embedding theorem 13
4. Formal normal forms 40
5. Holomorphic normal forms 61
6. Finitely generated groups of conformal germs 81
7. Holomorphic invariant manifolds 105
8. Desingularization in the plane 112

Chapter II. Singular points of planar analytic vector fields 143
9. Planar vector fields with characteristic trajectories 143
10. Algebraic decidability of local problems and center-focus alternative 159
11. Holonomy and first integrals 179
12. Zeros of parametric families of analytic functions and small amplitude limit cycles 200
13. Quadratic vector fields and the Bautin theorem 223
14. Complex separatrices of holomorphic foliations 232

Chapter III. Local and global theory of linear systems 255
15. General facts about linear systems 255
16. Local theory of regular singular points and applications 265
17. Global theory of linear systems: holomorphic vector bundles and meromorphic connexions 285
18. Riemann–Hilbert problem 312
19. Linear nth order differential equations 329
20. Irregular singularities and the Stokes phenomenon 351
Appendix: Demonstration of Sibuya theorem 365

Chapter IV. Functional moduli of analytic classification of resonant germs and their applications 373
21. Nonlinear Stokes phenomenon for parabolic and resonant germs of holomorphic self-maps 373
22. Complex saddles and saddle-nodes 404
23. Nonlinear Riemann–Hilbert problem 428
24. Nonaccumulation theorem for hyperbolic polycycles 442

Chapter V. Global properties of complex polynomial foliations 469
25. Algebraic leaves of polynomial foliations on the complex projective plane \( \mathbb{P}^2 \) 470
Appendix: Foliations with invariant lines and algebraic leaves of foliations from the class \( A_r \) 499
26. Perturbations of Hamiltonian vector fields and zeros of Abelian integrals 505
27. Topological classification of complex linear foliations 545
28. Global properties of generic polynomial foliations of the complex projective plane \( \mathbb{P}^2 \) 567

First aid 599
A. Crash course on functions of several complex variables 599
B. Elements of the theory of Riemann surfaces. 603

Bibliography 607
Index 621