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Preface and
Acknowledgements

We owe a special and deep debt of gratitude to E. Polishchuk (1913-1987), who awakened our interest in Hadamard’s life and mathematics when he invited us to participate in writing a book about the great French mathematician. The book “Jacques Hadamard”, by Polishchuk and Shaposhnikova written with participation of Maz’ya, was published in 1990, in Leningrad (now Saint Petersburg).

The biographical part of that book was very small, because original documents, letters and publications were inaccessible to us. Since our emigration to Sweden in 1990, we have been able to travel and consult new material concerning Hadamard. His personal papers disappeared during the war, and we have had to piece together the mosaic of his life from the odd fragments dispersed in archives and libraries in many different countries. The first part of the present book is a result of this search.

Professional historians of science have never focused their attention on Hadamard. During the thirty years after his death no attempt was made at writing his comprehensive biography. Meanwhile, there remain only a few of those who could share their reminiscences of the man who was once called the “living legend of mathematics”. As J.-P. Kahane remarked in his essay [II.29],¹ “no mathematical library contains the whole mathematical work of Hadamard, because – unlike the work of much less important mathematicians – it was never collected and published in its entirety. No street in Paris bears his name.² The legend needs a revival, especially in France.”

¹Roman numeration refers to one of four lists of references: I Bibliography of Jacques Hadamard, II Publications about Jacques Hadamard and his work, III General Bibliography, and IV Archival Material.
²Also there are no memorial plaques on the houses where Hadamard was born and died, and even Hadamard’s gravestone at the Père Lachaise cemetery does not have his name on it. (Auth.)
Although our exposition is documented, it does not pretend to be a deep historical study either. We neither could nor wished to attain this goal. Indeed, our aim was more modest: we merely wanted to tell a story of Hadamard’s life for professional mathematicians and undergraduate students with an interest in mathematics. It is Jacques Hadamard, his relatives, teachers, colleagues, friends and pupils who speak in the pages of this book.

In our endeavours to make the account of Hadamard’s life intelligible for a wider readership, we tried in the first part to obey the Justinian law dealing with “Malefactors and mathematicians and their like” which says: “The art of mathematics is also forbidden under pain of punishment” [III.88, p. 379], so we only mention some of Hadamard’s most important mathematical results, to give an idea why Hadamard is worth writing about.

We have described in more detail Hadamard’s contribution to mathematics in the second part of the book. Here the order of chapters reflects mainly the chronological order of his research interests. As before, we tried to address readers with rather modest mathematical background. We hope, nevertheless, that even an expert can find something of interest. Together with new material, Part II contains an extended and revised version of mathematical chapters of the Russian book [II.52]. Sections 11.3, 16.1 and excerpts of Chapter 9 reproduce with small changes the text left by E. Polishchuk, who died at the beginning of the work on [II.52]. Moreover, his notes were helpful when we wrote Chapters 10-12. We gratefully acknowledge his contributions to the present volume.

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