Introduction

The purpose and strategy of the *Phaenomena*

The *Phaenomena* is a geometrical treatment of some fundamental problems related to the risings and settings of stars and of important circular arcs on the celestial sphere. In fact, just over half its theorems (the last ten) are devoted to one of these problems, that of determining the length of daylight on a given day at a given locality, the two data on which the length of daylight obviously depends. Euclid’s is the earliest extant treatise dealing with this particular question. Theodosius (late second century BC) and Menelaus (end of the first century AD) also wrote treatises bearing on this problem: indeed, Neugebauer has described its solution as ‘one of the major goals’ of their spherical geometry.\(^1\) Some fifty years after Menelaus completed his *Sphaerica*, Ptolemy wrote his *Syntaxis mathematica* or *Almagestum* (*Almagest*); and the earliest record we have of an exact, geometrical solution to the problem of finding the length of daylight is in that work [cf. *Alm*. ii 9].

There were, however, earlier solutions. The oldest known method for solving this problem is found in a Ramesside papyrus of the twelfth century BC [Neugebauer 1975, 706], which contains approximate values for the lengths of daylight in each of the twelve months of the Egyptian year calculated on the basis of direct linear interpolation between a maximum value of 18 hours and a minimum of 6 hours. Babylonian solutions based on the same idea, but with different values for the maximum and minimum lengths of daylight, appear much later, an early specimen being contained

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\(^1\) The historical details given here and in the following paragraph may be found in Neugebauer 1975, 706–733.
in a tablet dating from about 400 BC [Neugebauer 1975, 709], still well before Euclid.

At some time, however, Babylonian astronomers devised a new method based on the insight that the length of daylight on a given day is the length of time it takes a certain arc of the ecliptic to rise over the horizon. More specifically, they realized that the length of daylight is the time it takes the semicircle of the ecliptic following the Sun to rise as the point on it occupied by the Sun moves across the sky from the eastern to the western horizon. This insight that the problem of finding the length of daylight could be solved by calculating the ‘rising times’ of ecliptic arcs led to an indirect solution according to which one determines the length of daylight in two steps:

(1) by assigning rising times, through calculation or the use of a simple linear scheme, to a set of consecutive arcs covering the ecliptic, say the individual signs, thirds of signs or even individual degrees, and then

(2) by computing the length of daylight on a given day as the sum of the rising times for the set of these arcs spanning the particular 180° of ecliptic arc rising after the Sun on the given day.2

Schemes of this type, which are intermediate between the earlier arithmetic solutions and the elegant trigonometric solutions of Ptolemy, rely—implicitly or explicitly—on symmetries which Ptolemy later proves, in order to obtain the rising times used in (1) above. Unlike Ptolemy’s solutions, however, such schemes employ not trigonometrical but arithmetical methods. These schemes were known to Greek writers of the Hellenistic period, as the Anaphoricus (On Risings) by Hypsicles of Alexandria, who probably wrote before 150 BC, attests. In fact, it is our view that Euclid knew of these older arithmetic approaches to the problem of determining rising times, and that one of his goals in writing the Phaenomena was to demonstrate geometrically the assumptions behind this arithmetic method.

2 Babylonian texts express such times in units of uṡ, each of which was \( \frac{1}{360} \) of a twenty-four hour period, that is, four minutes. Since 360° of the equator also rise at a uniform rate over the horizon in the same period, an uṡ is, in fact, the time it takes any one-degree arc of the equator to rise. For this reason they were called time degrees by the Greeks [see Neugebauer 1975, 367] and were used by Ptolemy in his table of rising times.

3 Unfortunately, the two texts in which such schemes occur (200 and 200b in Neugebauer 1955, i 87 and 214) are not dated, but they are part of a Babylonian archive of which the bulk of the material is datable to the middle of the second century BC [see 1955, i 9–11]. However, as Neugebauer emphasizes, it is futile to try to date the method from the text, which can only give a latest possible date.
Euclid organizes his exposition as follows:\(^4\)

Introduction:

1. Arguments for the sphericity of the cosmos and its uniform rotation about an axis. (The sphericity of the Earth is tacitly assumed.)
2. Definition of the principal celestial circles of interest to astronomers: meridian, tropic circles, arctic circle, horizon, equator, and ecliptic, together with a demonstration that the last three are great circles.

Preliminary topics:

3. Argument that the Earth is at the centre of the cosmos (Proposition 1).
4. Determination of when the ecliptic and meridian or ecliptic and horizon will be mutually perpendicular (Proposition 2).

On the risings and settings of stars:

5. A fixed star that rises and sets does so always at the same point on the horizon (Proposition 3).
6. Comparative order of risings and settings of stars located on great circles intersecting or not intersecting the arctic circle (Propositions 4 and 5).
7. Diagonally opposite stars on the ecliptic or equator rise and set in tandem (Proposition 6).

On ecliptic arcs, where they rise and how long they take to rise:

8. The arcs of the horizon where the whole ecliptic or individual signs rise (Propositions 7 and 8).
9. Comparison of the rising times of two given semicircles of the ecliptic (Proposition 9), and the following is either stated or trivially deducible: Daylight is longest when the Sun is at the beginning of Cancer; shortest when it is at the beginning of Capricorn. For any other sign in Table 1, the length of daylight when the Sun is at its beginning is less than it is when the Sun is at the beginning of the signs above it. Moreover, the length of daylight is the same when the Sun is at the beginning of signs side by side in this table.\(^5\)

Next, Proposition 10 allows one to pass from a comparison of the rising times of semicircles to a comparison of the rising times of smaller arcs.

10. Demonstration that the rising (setting) time of one arc is equal to the setting (rising) time of the other if either

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\(^4\) See English Glossary for definitions of technical terms.

\(^5\) See Figure 1 for a diagram of the ecliptic as viewed from its north pole. On the ecliptic, see also pp. 29–30 below.
Cancer
Leo     Gemini
Virgo   Taurus
Libra   Aries
Scorpio Pisces
Sagittarius Aquarius
Capricorn

Table 1.

(a) the two arcs are opposite with respect to the centre of the ecliptic, or
(b) if they are equidistant from a tropic (Proposition 11 and the lemma following Proposition 13).

Proposition 12 then compares the setting times of signs in the semicircle following Cancer,\textsuperscript{6} and Proposition 13 does the same for rising times in the semicircle following Capricorn.\textsuperscript{7} Propositions 12 and 13 taken together also assert the equality both of rising and setting times for arcs symmetrically situated with respect to the equator. Thus, if one places signs with equal rising and setting times opposite each other in a table one gets:

Capricorn     Gemini
Aquarius      Taurus
Pisces        Aries
Sagittarius   Cancer
Scorpio       Leo
Libra         Virgo

Table 2.

Figure 1, in which vertical lines join signs with equal rising times, and horizontal lines join the beginnings of semicircles with equal rising times, summarizes the information Euclid provides on the problem of rising times.

Conclusion:

11. Propositions 14–18 compare the times it takes equal arcs of the ecliptic to cross the visible and invisible hemispheres. Since the Sun is not a mathematical point but occupies about 1/2\textdegree of the ecliptic, and also

\textsuperscript{6} Since the Sun enters Cancer at the summer solstice, the rising times of these signs add up to the length of daylight on the longest day of the year.

\textsuperscript{7} The Sun enters Capricorn on the shortest day of the year, the winter solstice.
moves during the course of a daytime about $1/2^\circ$ (on the average) along the ecliptic (counterclockwise in Figure 1), it follows that the length of daylight is precisely equal to the time it takes that arc of the ecliptic occupied by any part of the solar disk during the course of daylight to cross the visible hemisphere.

One observes in this brief account that, beyond stating the equality of rising or setting times for arcs symmetrically situated with respect to the equator, Euclid does not compare rising times of arcs in the semicircle following Cancer nor (correspondingly) setting times in the semicircle following Capricorn. The reason is that no simple inequalities, like those for the cases he does discuss, apply here. To see this, consider *Alm.* ii 8 in which Ptolemy gives the following values for the rising times of the signs of Cancer, Leo, and Virgo,\(^8\) as computed for various localities that are classified according to maximum hours of daylight [see Table 3]. Thus, for localities near the equator, Cancer is the sign that takes longest to rise; for more northerly localities, it is Leo.

As Pappus of Alexandria (*fl. ca. 300 AD*) pointed out in his commentary on the *Phaenomena*, Euclid could not have stated inequalities corre-

\(^8\) The table indicates *inter alia* that for a locality in which the maximum length of daylight is $14^{1/2\text{ h}}$ (in Rhodes, for example), an arc of the equator measuring $36^\circ 28'$ rises while Virgo does. Since $1^\circ = 4$ minutes of rising time, this may be understood to mean that Virgo takes $2^\text{ h} 25' 52''$ to rise above the horizon.
sponding to Proposition 13 for signs in the semicircle following Capricorn [cf. Coll. vi 108 et seq.]. According to Pappus [Coll. vi 55], Hipparchus (fl. ca. 150 BC) was, in fact, the first to show this by numerical examples; but it was Menelaus who finally solved the problem left open by Euclid’s Phaenomena, in his own treatise on the setting times of arcs in semicircles of the ecliptic following Capricorn.

Pre-Euclidean works on the subject of the Phaenomena

A reading of the Phaenomena will demonstrate convincingly what the above list of topics suggests, namely, that Euclid was not the earliest writer on these topics. The sophistication of many of the propositions that Euclid assumes about the geometry of circles on the sphere [see pp. 19–27, below] indicates that he had a text or texts on that subject which we, following the Greeks, shall call spherics.

This observation, however, introduces two puzzles. The first is that, since the sole motivation for the abstract, geometrical propositions Euclid assumes is, apparently, their use in astronomy, there must have been, before Euclid’s time, a development not only of a geometrical theory of spherics but also of its use in the study of the astronomical problems motivating it. There is evidence of this pre-Euclidean geometrical theory of spherics in Euclid’s tacit assumptions and in Theodosius’ Sphaerica; but no trace remains of its earlier use in astronomy. The treatises by Autolycus, De sphaera quae movetur (On a Moving Sphere) and De ortibus et occasibus (On Risings and Settings), do not solve this puzzle but only emphasize it, since they too use some of the same geometrical theorems without any comment. Since, unlike Euclid’s floruit, which is traditionally taken to be around 300 BC, Autolycus’ floruit can be assigned with some assurance to

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9 On the goal and content of the former treatise (hereafter, De sphaera), see Berggren 1991.
333–300 BC [see Mogenet 1950, 5–9], one has an approximate latest date for the development of the requisite geometrical theory of the sphere.

This puzzle, however, turns out to be more apparent than real if one assumes that Euclid’s *Phaenomena* superseded the earlier treatments of the basic phenomena of the celestial sphere. This would parallel the case of his *Elements*, another work whose success meant the disappearance of earlier works on the subject. This seems to have been a common fate of early scientific works; indeed, it appears that Euclid’s *Conics* suffered the same fate when that of Apollonius appeared. So there is nothing at all unusual about certain strata of ancient works disappearing without a trace; in fact, one often feels that what has to be explained is not the disappearance of ancient works but their preservation.\(^\text{10}\)

The other puzzle, however, is real: one must face the question of who was responsible for the pre-Euclidean mathematical astronomy. A favorite candidate has been Eudoxus of Cnidus, a contemporary of Plato, whose contributions to mathematics were mentioned by Archimedes. In fact, among Eudoxus’ works on astronomy was one with the promising title *Phaenomena* (the first reported work with that title) and another called the *Enoptron*.\(^\text{11}\) According to Hipparchus, the two works agreed ‘very much with each other in practically all things’ [cf. Manitius 1894, 8.15–18].

However, when one examines the available evidence concerning the contents of these two books for material resembling that in Euclid’s *Phaenomena*, one finds very little. Since Hipparchus, in the tradition of Hellenistic scholarship, quotes Eudoxus primarily to disagree with him, it is difficult to get a conception of Eudoxus’ work as a whole. It is clear, however, that these books contained an extensive description of the constellations on the celestial sphere and their placement relative to each other, with descriptions of this sort:

> Between the Ursae lies the tail of Draco, having its outermost star over the head of Ursa Major. It curves by the head of Ursa Minor and then stretches under its feet. Making another turn here again it rears its head up and holds it forward. [Manitius 1894, 8.22–10.4]

Eudoxus also describes the circle of the winter tropic in these words:

> On it are the middle parts of Aries, the feet of Aquarius . . ., the mast of Argus, the back and chest of Centaurus, Lupus and the

\(^{10}\) The loss of Eudemus’ writings, among which was, apparently, a *Historia astrológiae* (*History of Astronomy*), is to be regretted.

\(^{11}\) An ἐνοπτρόν is something one looks into, such as a mirror; the word may thus signify a work that was an image in prose of the cosmos.
stinger of Scorpio. Then it [the circle] joins up to the middle of Aries through Sagittarius. [Manitius 1894, 22.1–9]

Admittedly there are in Eudoxus’ astronomical writings statements about circles on the celestial sphere of a more mathematical character. Thus, in his Enoptron, Eudoxus asserts that the tropic circles are cut by the horizon (of his locality) in a ratio of 5:3 [Manitius 1894, 22.19–22], and in his Phaenomena he gives the slightly larger ratio 12:7 [Manitius 1894, 28.8–13]. More importantly, Simplicius (sixth century AD), in his commentary on Aristotle, Meta. A 8, ascribes to Eudoxus a work called De celeritatibus (On Speeds) containing one of the earliest attempts to explain the principal phenomena of planetary motion using nested concentric spheres each rotating on an axis whose poles are carried on the next sphere out [cf. Heath 1913, 194]. The title and available evidence of the contents of this treatise, however, argue against its having contained the sort of considerations we find in Euclid.

There is, in conclusion, no evidence that Eudoxus wrote either on spherics or its astronomical application; nor are there any real clues as to when or by whom the theory was developed. We have argued elsewhere [cf. Berggren 1991] in support of an argument in Goldstein and Bowen 1983, that the development took place in the interval between the lifetimes of Eudoxus and Autolycus, but more than that we cannot say.

The integrity and authenticity of the Phaenomena

Two serious questions that must be addressed are whether Euclid wrote the Phaenomena and, if he did, whether what he wrote included the introduction. As for the first question, the ancient commentators strongly support the opinion that Euclid did write this work; and from the time of Galen onward they give details about the contents which make it certain that the work being referred to is in fact the present one. The earliest secure ancient testimony to the Phaenomena occurs in the writings of Galen, who was born probably around 130 AD, and wrote in his De placitis Hippocratis et Platonis (On the Doctrines of Hippocrates and Plato):

Euclid showed in Theorem 1 of the Phaenomena in a few words that the Earth is in the midst of the cosmos, as a point or a centre, and the students trust the proof as if it were two and two is four. [cf. Müller 1874, 655.8]

The next references are found in Pappus’ Coll. vi, which contains not only an extensive collection of lemmas to Euclid’s Phaenomena but also
references to Propositions 2, 11, 12 and 13. During the sixth century, Marinos of Neapolis, in his commentary on Euclid’s Data, refers to Euclid as the author of the Phaenomena; and John Philoponus in his commentary on Aristotle’s Physics remarks that Euclid’s Phaenomena is more ‘physical’ than Autolycus’ astronomical writings because Euclid mentions not only ‘motion’ but ‘substance’ as well, by which he must mean Euclid’s references to the Earth and the stars. Finally, there are the anonymous (and undated) scholiasts to Theodosius’ De diebus et noctibus (Days and Nights), who refer to Proposition 16 of the Phaenomena, as well as the scholiasts to Autolycus’ De ortibus et occasibus, who mention Propositions 13 and 14 of the Phaenomena.

Given the tight mathematical structure of the work, the ancient references to specific propositions tend to support the authenticity of the other propositions that either furnish the prerequisites for the arguments in the propositions referred to in the testimonia or complete the theory that these propositions address. Indeed the work forms a logical whole, whose coherence is evident on reading it: Euclid starts with basic matters that are easily demonstrated, and progresses in a connected way to rather special topics requiring more sophisticated proof.

But what of the style and structure of the work? Are these such as to give us confidence that the work was written by Euclid? That the structure of the proofs differs only in one small respect from that of the proofs in the Elements, may not count for much since it is difficult to be positive about how much the style and structure of either work owes to later editors; and there is no reason to assume that Euclid wrote all his works, which cover such a wide range of topics, in a single style. And it seems, at the very least, that there is no reason to deny that the work is by Euclid on the grounds of structure in the main body of the treatise.

However, when one considers the larger framework of the work some doubts arise. From a modern point of view, rigorous proofs depend on clearly stated definitions and postulates. One does not demand perfection in this regard; but in other works such as the Elements and the Optics, Euclid showed himself aware of the need for this accompanying appara-

12 For details, see Menge 1916, xxxii–xxxiv.
13 Thus, one has the same ‘setting out’ of the theorem to be proved, first in general terms and then with reference to parts named in a specific diagram, followed first by any necessary auxiliary constructions and then the proof proper, which ends with a statement of what has been proved, first in terms of the diagram and (sometimes) in general terms. (In the Elements, theorems, as opposed to problems, invariably end with the beginning of a general restatement of the theorem followed by the words ‘and the rest?’)
EUCUR’S PHAENOGENA

tus. In both these works the postulates are idealizations of experience that form the initial statements in a deductive system, and in both he states them without further comment. In particular, the Optics opens with seven statements labelled ‘definitions,’ some of which are, however, clearly postulates.

In contrast, the Phaenomena opens with a discursive introduction in which we find a mixture of definitions and statements that it is said must be assumed, including:

1. the stars are set into the surface of a single body and are carried around on circles which are everywhere equidistant from the eye;
2. the circles are all parallel, centering around a fixed pole star; and
3. the cosmos is spherical.

And the mixture of observations and loose geometrical arguments introducing these statements is reminiscent of what one finds in an extant edition of the Optics, where some unknown editor later attached to the introductory ‘definitions’ an account of what Heiberg [1882, 138] has described as a lecture on the Optics. In it the lecturer attempts very much the same kind of thing as the author of the introduction to the Phaenomena, that is, to justify the basic principles by reference to everyday observations. These similarities lead one to suspect that the Phaenomena suffered the same treatment as the Optics.

Indeed, Neugebauer [1974, 756] has attacked the authenticity of the introduction to the Phaenomena on similar grounds, namely, that a treatise of a similar character, Theodosius’ De diebus et noctibus, has an introduction and definitions which are only scholia, as is the case with the definition of uniform motion in De sphaera. Neugebauer also argues that much of the material in the introduction either repeats what is in the treatise or is irrelevant to it [see p. 51n33, below].

It should be said, however, that regarding the Optics and De diebus there can be no doubt that the introductions are spurious, given that the writer of the introduction to the Optics refers to the author in the third

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14 The introduction to one of the extant recensions of the Optics is clearly not by Euclid. Heiberg assumed on insufficient grounds that it is by Theon, the industrious editor of the fourth century AD; but this is not widely accepted today.

15 Defined are some common astronomical terms, such as ‘ever-visible,’ ‘equator,’ ‘horizon,’ ‘meridian,’ and ‘tropics,’ as well as such special notions as ‘time of revolution of the cosmos’ and ‘the passage of an arc across the visible or invisible hemisphere.’

16 ‘Observation’ must be understood not in a technical sense but vaguely, as in ‘We’ve all seen this phenomenon, haven’t we?’
person, and the writer of the introduction to De diebus refers to Theodosius; whereas there is no such clear evidence in the Phaenomena. Moreover, according to Aristotle’s view of science, the introduction is more congruous with the rest of the Phaenomena than has heretofore been observed.\textsuperscript{17} For, the terms θετέον and ὑποκείσθω, which the writer of the introduction uses to introduce the special assumptions of the work, remind one of the Aristotle’s vocabulary in the Analytica posteriora; and, indeed, much in the Phaenomena is reminiscent of this work.

To begin with the vocabulary, in his Analytica posteriora Aristotle specifies two kinds of ultimate premises in any given science. Some are axioms (ἀδιάφορα)—called common notions (κοιναὶ ἐννομαὶ) in Euclid’s Elements—which contain principles used in demonstrations in all sciences. Others are θέοεις or principles peculiar to the science in question. These are either assertions that one of two mutually exclusive alternatives is the case (ὑπὸ-θέοεις) or definitions (ὅραμαὶ). In the introduction to the Phaenomena, it is precisely the principles peculiar to this treatise that are introduced with the word θετέον.\textsuperscript{18} They are not principles of mathematics, but of astronomy; and, indeed, Aristotle specifically mentions in An. post. i 13 that the science of the phaenomena is subsidiary to astronomy, this latter term being used indifferently to denote either mathematical astronomy or the practical sort sailors used. In this ranking of the sciences, it is the task of the lower science—in this case, the science of the phaenomena—to establish the facts and that of the higher science (mathematical astronomy) to establish the reasons for these facts.

Seen from this perspective, then, the available evidence suggests that Eudoxus’ Phaenomena was concerned with stating certain facts of the case, and that Euclid’s Phaenomena was concerned with establishing the reasons why the facts must necessarily be as they are. And the reasons are established in Euclid’s work just as Aristotle prescribes, by demonstrations from the first principles (including the θεοεις) which show that, on the basis of such principles, the facts (in this case, the phenomena) are necessary.

How are the first principles to be established? At the end of An. Post. ii, Aristotle states that they are arrived at by repeated visual sensations which leave their marks in the memory. We then reflect on these memories and arrive, by a process of intuition (ὡς), at the first principles. Now, this

\textsuperscript{17} The authors are indebted to Henry Mendell in the following remarks about Aristotle’s An. post.

\textsuperscript{18} This is a form of the verb τίθημι, from which the Aristotelian θέοεις is also derived. The other word used in the introduction for assuming something, ὑποκείσθω, is a form of the verb ὑπόκειμαι which, according to Liddell and Scott, is used in Greek literature as the passive of ὑποτίθημι, a compounded form of τίθημι.
is what happens in the introduction to the *Phaenomena*, when the author justifies his *θέσεις* in terms of sensations whose memories he asks the reader to recall.

Thus, both in its vocabulary and evident purpose, the introduction to the *Phaenomena* fits well with the rest of the treatise. For, it complements the demonstrations contained in the propositions by arguing for the hypotheses on the basis of the phenomena (and by providing as well Aristotle's other sort of *θέσεις*, namely, the definitions) and then by showing how these work together to give the reasons for the phenomena.

Having said this, however, we should emphasize that the possibility of reading the treatise in Aristotelian terms does not imply that the writer was consciously following Aristotle's *dicta*. Indeed, R. McKirahan [1992] has recently argued that the mutual interaction between philosophic strictures and Greek mathematical practice was a complex one, and that the streams of influence by no means flowed in only one direction. Aristotle was working at a time of active mathematical practice, and his writings reflected as well as influenced that practice.

We should also emphasize that our observation that the introduction fits well with the treatise is not intended to settle the question of the introduction's authenticity. A suggestion about how a treatise might be read should not be construed as a historical argument for the authenticity of all its parts. It implies nothing stronger than the conclusion that, if the treatise were expanded by the inclusion of the introduction, it was done by someone who knew how to do it in a way that made sense from a philosophic (and perhaps pedagogic) viewpoint.

In the end it is not easy to decide on the authenticity of the introduction. However, we think that, on the balance of the evidence, the introduction did not belong to the treatise originally as written by Euclid. Authentic introductions to Hellenistic mathematical treatises from the time of Euclid to that of Apollonius have the form of letters.\(^{19}\) Moreover, the *Phaenomena* became part of a corpus of material [see pp. 15–16, below] whose pedagogic aim rendered it a likely candidate for the addition of an introductory commentary, as Neugebauer pointed out happened in the case of some other members of this corpus. Moreover, the principal ancient commentator on the treatise, Pappus, makes no mention of the introduction, despite its length and the fact that it would have provided ample material for a writer

\(^{19}\) An apparent exception to this statement is the introduction to the work on music theory, the *Sectio canonis*, which is not in the form of a letter, although the work has traditionally been included in the Euclidean canon. However, both editors (Menge and Jan) of the text of the *Sectio canonis* have doubted its attribution to Euclid in its present form.
like him to comment on. To go further towards resolving the doubts raised above one has to consider the history of the text and what it reveals about the origins of the present edition of the *Phaenomena*.

History of the text

Prior to H. Menge's publication of the *Phaenomena* in 1916, the best Greek text of the work was David Gregory's edition and translation of the complete works of Euclid published at Oxford in 1703. Of this edition, however, Heiberg [1882, 47] wrote:

Now it strikes the eye immediately that the *Phaenomena*, in the form in which Gregory edited it, has been much marred by additions. For four of the eighteen theorems (6, 12, 14 and 15) other proofs are present, of which that for theorem 6 is indirect (which in this treatise is otherwise never the case) and the second proofs for 12 and 14 deviate in reality too little from the first for the two [proofs for each theorem] to be by the same author. Also there are, besides one denoted as such in the text (to Proposition 14), four more scholia in the text, which by the note 'moreover scholion' identify themselves clearly enough as trimmings (one to Proposition 12 and three to Proposition 14).

Gregory based his edition of the *Phaenomena* on a number of manuscripts; but, as with all manuscripts of the *Phaenomena* known at the time, they were members of a single family, whose oldest representative is the codex Vaticanus graecus 204. Indeed, Heiberg wrote his *Litterargeschichtliche Studien* [1882] in part to call the attention of the scholarly world to the new information about the history of Euclid's works to be gained from a study of cod. Vind. gr. 31.13. In view of the merits of this codex and its relatives, all of which represented a single family, Menge denoted the version of the *Phaenomena* found in this family by the siglum a, and used b to denote the version found in the family best represented by cod. Vat. gr. 204.

There is no reason why Euclid should not have issued two editions (as they would now be called) of a work that is so clearly not the last word on its subject. (This is not to say that there are no detectable accretions, some of which are sufficiently extensive to merit inclusion in an appendix.

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20 This family consists of some mss of texts of the Little Astronomy, described below; and its classification was worked out in Mogenet 1950. (Mogenet was concerned primarily with the mss of Autolycus, but his results should also be valid for the *Phaenomena*.)
of ten pages to Menge's edition.) However, none of the features of b, which we shall discuss below, suggests anything other than work by a later editor; and no writer on the subject has suggested that b represents Euclid's reworking of a.

Heiberg called the recension a 'far nearer to the original' than b because the scholia are separated from the text and the superfluous alternate proofs are not present. Heiberg also emphasized that the variations between the proofs in a and b ensure that neither a nor b is a source for the other.21

In our opinion, however, Heiberg overstates the conceptual differences in the proofs, for they amount only to the following. In Proposition 10, despite Heiberg's claim that the proof differs in conception, the only difference is that the author of b, misled perhaps by a diagram, has assumed unnecessarily that point A on the ecliptic must be the beginning of the sign Cancer. In Propositions 11 and 12 the differences, apart from details, lie in the fact that in a both have for the proof of their last parts only 'Similarly we shall prove that...' As for the proof of the Lemma—labelled 14 in certain manuscripts of a and appearing as a scholium in b—Heiberg again overstates the difference. While it is true that a proves the rising time of one arc is equal to the setting time of the other and b proves the setting time of one equal to the rising time of the other, there is no difference in the ideas behind the proofs. Finally, the proof in a of Proposition 16 (17 according to the numbering in a) lacks the necessary 'Similarly we shall prove...'; found in b.

The presence of the full proofs of Propositions 11 and 12 in b is certainly consistent with the hypothesis that an editor has supplied details for a case that the author thought the reader could work out for himself (or that the teacher could work out for him). The misunderstanding in the proof of Proposition 10 might well be a sign of a careless editor; and, in general, none of the differences are such as to suggest that the two versions represent different editions of the work done by Euclid himself. That all manuscripts

21 Heiberg's detailed comparison of other differences between a and b is as follows [1882, 50–51]:

Through Proposition 8 they are essentially the same, and even in Proposition 9 the differences have the character of being only different readings. In 10 the proofs differ in conception and in 11 the proofs are quite different in detail. As well, in 11 the second half [in b] is replaced by a short sentence beginning 'Similarly!' The proofs of 12 differ in form. In 13 the proofs differ not only in approach but also in the lettering of the figures. The Lemma is numbered as a proposition, and has a proof different in conception. [In a] the proof of Proposition 14 (numbered 15) is much abbreviated, and the manuscripts break off shortly after the first part of the proof of Proposition 16.
of a break off midway through Proposition 16, argues that they all stem
from a common archetype which may have lacked a page or been damaged
at the end of a roll. The incomplete state of a would also explain why there
are many fewer copies of it than of b, since a copyist would naturally pick a
complete version if one were available.

Although Heiberg is surely right in insisting that cod. Vind. gr. 31.13
presents a version of the Phaenomena nearer the original than the text
Gregory printed, this codex is by no means a faithful copy of the original
edition, even if one puts aside the fact that it lacks Propositions 17 and 18
as well as half the proof of Proposition 16. Thus, in his commentary on
the Phaenomena, Pappus [Coll. vi 104–129] has extensive notes on various
points in the work; and in vi 104 he remarks that in Proposition 2 Euclid
omitted any demonstration of the case when the zenith is on the tropics or
between them. But all known manuscripts of the Phaenomena, including
the Vienna codex and its relatives, discuss this case in Proposition 2; and,
although it is possible that other copies of the text at Pappus’ time had
a discussion of the missing cases, it is simpler to assume that in Pappus’
time there was still only one version of the treatise, and that sometime
later an editor completed Proposition 2 by including the cases mentioned
by Pappus.

By the time of Pappus, the Phaenomena had become part of a group of
texts that provided a student versed in the requisite material from the Elements with the additional information necessary for the study of Ptolemy’s
Almagest. Indeed, the purpose of Pappus’ Coll. vi was to comment on this
material, referred to by an anonymous scholiast as the Little Astronomy.
Since Pappus also comments on Autolycus’ De sphaera, Euclid’s Phaenom-
ena and Optics, Aristarchus’ De magnitudinibus et distantia solis et lunae
(On Sizes and Distances of the Sun and Moon), Theodosius’ Sphaerica and
the same author’s De diebus et noctibus, we assume that the Little As-
tronomy included at least these six works. However, it seems to have come
to include other works as well, since, for example, the three codices Vat.

22 On this hypothesis, the citation in Theodosius’ Sphaerica of text that is unique
to b would be a later interpolation in Theodosius’ work. This would not be at all
surprising, since Hellenistic mathematical writers, as opposed to commentators,
seldom cite their predecessors within the confines of a mathematical treatise.

23 If there were multiple versions prior to the time of Pappus, one would have
to assume either that he did not know the ones which were the archetypes of
our present versions and which bore the complete form of Proposition 2, or that
emendations were made in different archetypes after Pappus’ time so that they
would all agree in the wording of Proposition 2. Neither of these hypotheses
seems to us to be very likely.
gr. 191, 202, and 204 all contain, in addition to the above six works, three other treatises: Autolycus’ *De ortibus et occasibus*, Hypsicles’ *Anaphoricus*, and Theodosius’ *De habitationibus* (*On Habitations*)—all dealing with very much the same sort of problems as the six treatises Pappus comments on. (And two more codices, Vat. gr. 203 and Parisinus gr. 2364, lack only the two treatises by Euclid which, in any case, we know from Pappus belonged to the collection.)

Hence, despite Neugebauer’s reservations [1975, 768–769] about the existence, or importance, of such a collection, Mogenet [1950, 163–166], whose evidence we have summarized above, presents good reason for accepting as a reasonable interpretation of the evidence the thesis that a body of teaching material later known as the Little Astronomy was formed by at least the fourth century AD.

The history of the *Phaenomena* cannot be considered apart from the role of the Alexandrian commentators in mathematical education and, in particular, the history of the the Little Astronomy. For, an editor interested in using Euclid’s *Phaenomena* for teaching would have found several problems with the work as Euclid wrote it. Its tacit use of non-trivial theorems of sphericity and its use of such non-trivial astronomical notions as rising times without any explanation indicate that the treatise was originally intended for a learned readership and not as a text for beginners. Thus, its inclusion in the Little Astronomy necessitated not only a series of lemmas by Pappus but further changes as well in order to make it more suitable for instructional purposes. And among these changes was the addition of an introduction by the editor, one which would lead the reader gently into the abstract considerations of the text proper.

In seeking out a possible candidate for editor, Menge did not think it necessary to look much past Pappus’ time, since Menge believed that Theon of Alexandria, who lived only about forty years after Pappus, edited not only Euclid’s *Elements* and *Data*, but his *Optics* as well. Indeed, Menge, who was obviously aware of the problems Euclid’s treatise raises, went so far as to state that it may have been for the purposes of inclusion in the Little Astronomy that Theon prepared the new edition, b, of the *Phaenomena*.24 (In that case, if we accept Heiberg’s contention that a is the earlier version, it was presumably Theon who incorporated into the text those scholia that are clearly separated from it in a and added the four alternate proofs.) He wrote that Theon’s name ‘comes to mind’ as a possible compiler of b because Theon edited Euclid’s *Elements*, *Data*, and *Optics*.

24 Menge thus appears to echo Heiberg [1882, 51], who said that one is justified in ascribing the variants in b that are not simply copying errors to a later reworking that was probably undertaken for the Little Astronomy.
First of all, however, if Theon produced \( b \), it is difficult to determine who was the editor of \( a \). In our view, \( a \), which we take to be the earlier version, had not been prepared by the time of Pappus. There is, after all, no evidence in Pappus’ text that any of the alternate proofs found in \( a \) and incorporated into the text of \( b \) had yet been framed. If we are right, then, there is too short an interval between Pappus and Theon to produce edition \( a \), with its alternate proofs and introduction, in time for Theon to rework it into \( b \). And there are no likely candidates for the editor of \( a \) in the few decades between Pappus and Theon.

It seems to us more likely, therefore, that Pappus’ commentary inspired the work which culminated in a later writer’s producing version \( a \), which represents Euclid’s text after an early attempt at editing it; and that \( b \)—all of whose material can reasonably be seen as a reworking of that in \( a \)—represents a ‘fleshed-out’ version of \( a \).

Furthermore, Menge’s proposal that Theon edited the \textit{Phaenomena}, quite apart from the question of which version, should be treated with the greatest caution. That Theon edited the \textit{Optics} is, as we have noted, simply Heiberg’s assumption. And the \textit{Elements} and the \textit{Data} are of a very different sort from the \textit{Phaenomena}; thus, one can well imagine a mathematician’s being interested in them without necessarily being interested in the \textit{Phaenomena}.

On the other hand, Theon’s interest in astronomy is evident from his commentaries on most of Ptolemy’s \textit{Almagest} and on his \textit{Canones manuales} (Handy Tables). And even though a commentary would have been a very different thing from an edition of the \textit{Phaenomena}, it does establish that Theon was deeply involved in the study and teaching of astronomical material. Moreover, Theon is the only individual we know of after Pappus who is demonstrably associated with any substantial work on the Little Astronomy. In fact, the anonymous author of a study of isoperimetric figures [see Hultsch 1887–1888, 1142] cites him as the author of a commentary on the Little Astronomy. Finally, the dates are right. As we have already noted, there are grounds for maintaining that the \textit{Phaenomena} was first edited after Pappus, who wrote half a century earlier than Theon. Thus, in our view, Theon may have been the source of version \( a \) of the \textit{Phaenomena}.

In conclusion, we propose that Pappus had a version of the \textit{Phaenomena} in substantially the form in which Euclid wrote it, and that he then composed the notes to aid in its teaching that are found in his \textit{Coll. vi}. At some later time, an editor wrote an introduction to the work, changed Proposition 2 to take into account Pappus’ notes, and attached some alternate proofs which had appeared after Pappus’ time and perhaps some further additions to produce what was essentially \( a \). Still later, the text was further
reworked to incorporate the alternate proofs and some scholia, resulting in our version b. Subsequently, the end of the earlier version was lost; and so b became the more frequently copied edition, even though enough of the earlier remained for it to be worth copying and (hence) preserving as our version a.