Introduction

Magic is a world unto itself. Thousands of people have come under its spell. They are organized into societies at the local, national, and international levels. There are magic competitions and publications on magic. Of course, there are wide differences in the degree of professionalism and originality among magic aficionados.

Since no one is able to perform actual magic, it is necessary to create somehow the illusion that something impossible has taken place. Therefore, dexterity and deceptions prepared in advance play a large role in traditional magic: pretending to pass a coin from one hand to the other, boxes with hidden drawers, hiding a coin or playing card inconspicuously in the hand (“palming”), and much more.

In this book, it is mathematics that does the heavy lifting. This is possible because based on the proper mathematical background, many useful patterns and a great deal of information can be discovered in regard to which those ignorant of the underlying mathematics cannot imagine that one can make successful predictions.

For example:

- Someone thinks of a number, which is then manipulated in certain ways: reverse the order of the digits, add a certain number or multiply by some number, etc. And one can predict the final result, which is independent of the number chosen.
• A deck of cards is shuffled, and it appears as if the order of the cards has been completely randomized. Nevertheless, it is possible to determine the location in the pack of a preselected card.

For years I have been fascinated by this aspect of mathematics. Of course, I have attempted gradually to learn the “classical” parts of magic, but I’m afraid it’s going to be awhile before—for example—I incorporate in my shows cutting a rope in half and splicing it back together or suddenly changing the colors of a bunch of silk kerchiefs.

Those whose memories of studying mathematics in bygone school-days are not particularly pleasant may be worried that this book might contain a large amount of theory to wade through before we get to doing magic. Let me put your mind at ease at once: you can read this book as though it were nothing more than a standard introduction to the art of magic. There are complete descriptions of what you have to do to perform a magic trick successfully. And if you follow the simple directions, everything is guaranteed to work.

Nevertheless, it is my hope that many readers will want to know why a particular trick actually works. They will find extensive mathematical explanations.

Let me add that nowhere does it get really difficult. Anyone who can recall a few mathematical terms should have no difficulty: How do you multiply two three-digit numbers by hand? What are the divisors of a given number? What is a prime number?

Thus this book can be seen as an invitation to learn about some interesting and important aspects of mathematics: How do mathematicians conquer the infinite through induction? Why are the remainders from division important? What is coding? How do mathematicians deal with probability?

This book begins with an introductory section in which some basic terminology is explained. You may read it at once or worry about it only when the relevant explanations are needed in a particular magic trick. Following this section are four chapters in which magic tricks are presented. Each chapter is based on a different mathematical principle:
• Chapter 1 (“You Can Count on It”) describes tricks based on the counting numbers. There are some very simple sections as well as those that are more demanding. The first section requires, with few exceptions, only concepts learned in elementary school. In the second section, prime numbers and some of their surprising properties play an important role.

• In Chapter 2 (“Let’s Mix It Up!”), numbers are no longer at the forefront. It is much more about analyzing how certain information can be retained after a set of objects has been apparently hopelessly jumbled. Since this can be accomplished with playing cards using various shuffling techniques, this chapter will be primarily concerned with card tricks.

  The sections of this chapter emphasize different aspects of this idea depending on what sort of information one begins with: invariants (what properties of the cards remain unchanged under shuffling?), detective work (what can be predicted if you make a bit more effort?), and symmetry (what use can one make of the deck having certain symmetry properties before it is shuffled?).

• The basic idea of Chapter 3 (“Coding”) is encrypting information in such a way that it can’t be detected by an observer. We present here some simple tricks, as well as others for which the intellectual effort on the part of the presenters (magician and helper) is not insignificant.

• The last chapter, Chapter 4 (“Chance Makes Magic”), deals with probability. There are many surprising phenomena to be discovered in this area (so-called paradoxes). They are based on the fact that evolution has given the human species rather a poor intuition about probabilities.

  Some of these paradoxes will be used for magic tricks. We should, however, mention that things can go wrong, with the result that the desired effect does not occur. The probability of such an outcome is small, but it is in the nature of probability that one is never dealing with absolute certainty.
There is finally an appendix, in which those who are interested can read about details of two tricks with a somewhat more demanding mathematical background, and the book ends with a selection of recommended further reading. There one can find books that were useful to me in writing this book, general suggestions, and links to some articles on the topic of magic (with QR codes for simplicity in finding them).

As with all somewhat demanding human endeavors, it does not suffice to read only a single book. No one learns all by oneself how to play the piano, practice judo, or sail a boat—not even to tie one’s shoelaces. And this is true of magic as well. As is so often the case, practice makes perfect. Try out the magic tricks, and then work on them over and over, alone at first, and then when you have things going smoothly, you can appear before an audience.

And there is one other aspect on which you can continue to work. Magic tricks are like birthday presents: the packaging plays a very important role. In this book you will find many hints in this direction, but there is also a great deal of scope for individual creativity and variation. And it is also important to know what one should not do: never (really, never) explain the secret behind the trick you have just performed. Your audience will be more disappointed than impressed.

Let me close this introduction with a note to my female readers. In recent years it has become customary to recognize the equal status of women by writing “his or her” or “her/his” or perhaps “his” on one occasion and “her” on another. I shall attempt to balance gender equality with readability, with some “he” magicians and some that are “she.” And the same goes for the magician’s assistant and the participating members of the audience. But in any case, I want to say that my interest in female magicians is just as fervent as my interest in male magicians. Furthermore, I should note that today the field of magic is dominated by men. The number of female magicians is vanishingly small, and women are much more likely to appear as the magician’s “charming assistant” than as the star of the show. I hope that this book contributes to the amelioration of this inequality.