## Contents

Preface vii

Chapter 1. Basics on tensor norms 1
The algebraic preliminaries 1
1.1. Reasonable crossnorms, including the norms ∧ and ∨ 5
1.1.1. Definitions 5
1.1.2. Injectivity of ∨ and projectivity of ∧ 11
1.1.3. The universal mapping property of ∧⊗ and the dual of X ∧ Y 13
1.1.4. Examples: C(K) ∧ X and L₁(µ) ∧ X 14
1.1.5. Integral bilinear forms and the dual of X ∧ Y 22
1.2. Definition of ⊗-norms 25
1.2.1. Fundamental operations on ⊗-norms 26
1.2.2. Order relations among ⊗-norms 28
1.3. Extension of ⊗-norms to spaces of infinite dimensions 29
1.3.1. Metric accessibility and accessibility 32
1.4. Bilinear forms and linear operators of type α 40
1.4.1. General properties of α-forms 42
1.4.2. General properties of α-integral operators 47
1.4.3. Composition of α-integral and ∨-integral operators 48
1.4.4. Accessibility and metric accessibility (continued) 50
1.5. α-nuclear forms and operators 54
1.6. The Dvoretzky-Rogers theorem, Grothendieck-style 59
1.6.1. The fundamental lemma 59
1.6.2. Consequences 63

Chapter 2. The role of C(K)-spaces and L₁-spaces 67
2.1. Complements on ∧ and ∨ 67
2.1.1. Representability, equimeasurability and nuclearity 72
2.2. Fundamental linear topological properties of C- and L-spaces 76
Notes 81
2.3. Injective and projective ⊗-norms 84
2.4. Formation of new ⊗-norms 90
2.5. Complements on /∧, ∧\, /∧\, \∧, \∨/, \∨/ 101
2.6. A table of natural ⊗-norms 106

Chapter 3. ⊗-norms related to Hilbert space 111
3.1. Definitions and generalities about H and H* 111
3.2. Hermitian H-forms 119
3.3. Hermitian H*-forms 122
## CONTENTS

3.4. Basic relations between $H$, $H^*$, etc. 131  
3.5. The “little” Grothendieck inequality 132  
3.6. The classes of $\alpha$-integral operators between Hilbert spaces 142

Chapter 4. The Fundamental Theorem and its consequences 149  
4.1. Functions of type $\alpha$ 149  
4.2. The Fundamental Theorem (Grothendieck’s inequality) and its variants 152  
4.3. Consequences to the theory of linear operators 159  
4.3.1. Compositions of operators between spaces of type $C,L$ and $H$ 159  
4.3.2. Linear topological characterizations of Hilbert space 160  
4.3.3. A theorem of Littlewood 161  
4.4. A table of the fourteen natural $\otimes$-norms 163  
4.4.1. A summary with regards to the characterizations and factorization schemes of the various classes of integral operators 164  
4.4.2. There are at most 14 natural $\otimes$-norms 165  
4.4.3. There are exactly 14 natural $\otimes$-norms 166  
4.5. Notes and remarks on the complexification of tensor norms 168  
4.5. Notes and remarks on the natural tensor norms and Banach Algebras 168  
Further notes and remarks 175

Glossary of terms 177

Appendix A. The problems of the Résumé 183  
A.1. Problem 1: The approximation problem 183  
A.2. Problem 2: The possible reduction of the table of “natural” tensor norms 186  
A.3. Problem 3: Grothendieck’s inequality and the “best” constant 191  
A.4. Problem 4: Algebraic-topological properties of $C^*$-algebras 201  
A.5. Problem 5: Characterizing classes of spaces by the behavior of tensor products and the action of operators on the spaces 208  
A.6. Problem 6: Comparison of the projective and injective tensor products 209

Appendix B. The Blaschke selection principle and compact convex sets in finite dimensional Banach spaces 211  
B.1. Blaschke’s Selection Principle 211  
B.2. Compact sets in Euclidean spaces 212  
B.3. Ellipsoids in finite dimensional Banach spaces 216

Appendix C. A short introduction to Banach lattices 217  
C.1. The facts, ma’m, just the facts 217  
C.2. Some basics about duality in Banach lattices 218  
C.3. Lattice homomorphisms 222  
C.4. $AM$-spaces and $AL$-spaces 225  
C.5. Kakutani’s vector lattice version of the Stone-Weierstrass theorem 227  
C.6. Kakutani’s characterization of $AM$-spaces with unit 228  
C.7. $AL$-spaces: The Freudenthal-Kakutani theorem 229  
C.8. Kakutani’s characterization of $AL$-spaces 234
CONTENTS

C.9. Grothendieck's inequality for Banach lattices 235
Notes and remarks 240

Appendix D. Stonean spaces and injectivity 241
D.1. The Nakano Stone Theorem 241
D.2.Injective Banach spaces 242
Notes and remarks 253

Epilogue 255
References to operator spaces 259
Monographs 259
Papers 260

Bibliography 261
Author Index 271
Index of Notation 273
Index 277