Contents

Preface to the Series xi
Preface to Part 1 xvii

Chapter 1. Preliminaries 1
§1.1. Notation and Terminology 1
§1.2. Metric Spaces 3
§1.3. The Real Numbers 6
§1.4. Orders 9
§1.5. The Axiom of Choice and Zorn’s Lemma 11
§1.6. Countability 14
§1.7. Some Linear Algebra 18
§1.8. Some Calculus 30

Chapter 2. Topological Spaces 35
§2.1. Lots of Definitions 37
§2.2. Countability and Separation Properties 51
§2.3. Compact Spaces 63
§2.4. The Weierstrass Approximation Theorem and Bernstein Polynomials 76
§2.5. The Stone–Weierstrass Theorem 88
§2.6. Nets 93
§2.7. Product Topologies and Tychonoff’s Theorem 99
§2.8. Quotient Topologies 103
Chapter 3. A First Look at Hilbert Spaces and Fourier Series 107
  §3.1. Basic Inequalities 109
  §3.2. Convex Sets, Minima, and Orthogonal Complements 119
  §3.3. Dual Spaces and the Riesz Representation Theorem 122
  §3.4. Orthonormal Bases, Abstract Fourier Expansions, and Gram–Schmidt 131
  §3.5. Classical Fourier Series 137
  §3.6. The Weak Topology 168
  §3.7. A First Look at Operators 174
  §3.8. Direct Sums and Tensor Products of Hilbert Spaces 176

Chapter 4. Measure Theory 185
  §4.1. Riemann–Stieltjes Integrals 187
  §4.2. The Cantor Set, Function, and Measure 198
  §4.3. Bad Sets and Good Sets 205
  §4.4. Positive Functionals and Measures via $L^1(X)$ 212
  §4.5. The Riesz–Markov Theorem 233
  §4.6. Convergence Theorems; $L^p$ Spaces 240
  §4.7. Comparison of Measures 252
  §4.8. Duality for Banach Lattices; Hahn and Jordan Decomposition 259
  §4.9. Duality for $L^p$ 270
  §4.10. Measures on Locally Compact and $\sigma$-Compact Spaces 275
  §4.11. Product Measures and Fubini’s Theorem 281
  §4.12. Infinite Product Measures and Gaussian Processes 292
  §4.13. General Measure Theory 300
  §4.15. Another Look at Functions of Bounded Variation 314
  §4.16. Bonus Section: Brownian Motion 319
  §4.17. Bonus Section: The Hausdorff Moment Problem 329
  §4.18. Bonus Section: Integration of Banach Space-Valued Functions 337
  §4.19. Bonus Section: Haar Measure on $\sigma$-Compact Groups 342
<table>
<thead>
<tr>
<th>Chapter 8. Bonus Chapter: Hausdorff Measure and Dimension</th>
<th>679</th>
</tr>
</thead>
<tbody>
<tr>
<td>§8.1. The Carathéodory Construction</td>
<td>680</td>
</tr>
<tr>
<td>§8.2. Hausdorff Measure and Dimension</td>
<td>687</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 9. Bonus Chapter: Inductive Limits and Ordinary Distributions</th>
<th>705</th>
</tr>
</thead>
<tbody>
<tr>
<td>§9.1. Strict Inductive Limits</td>
<td>706</td>
</tr>
<tr>
<td>§9.2. Ordinary Distributions and Other Examples of Strict Inductive Limits</td>
<td>711</td>
</tr>
</tbody>
</table>

Bibliography                                                                   713
Symbol Index                                                                   765
Subject Index                                                                   769
Author Index                                                                   779
Index of Capsule Biographies                                                   789