Contents

Preface v

Chapter 1. Conventions 1
  1. Vector Spaces 1
  2. Algebras, Coalgebras, and Modules 4
  3. Dual Modules 8
  4. Bialgebras and Hopf Algebras 8
  5. Θ – Ξ Modules and Relative Homological Algebra 11
  6. Algebras with Coproducts 15
  7. Chains and Cochains 20
  8. Differential Algebras and Coalgebras 23
  9. Simplicial Θ-Modules 25
  10. Homology of Simplicial Sets and Simplicial Groups 29
  11. Cotriples, Simplicial Objects, and Projective Resolutions 35
  12. Simplicial Θ-Coalgebras and Steenrod Operations 37
  13. Steenrod Operations on the Cohomology of Simplicial Sets 40
  14. Steenrod Operations on the Cohomology of Hopf Algebras 41
  15. Bisimplicial Objects 44
  16. The Spectral Sequence of a Bisimplicial Θ-Module 45
  17. Cup-k Products and Bisimplicial Θ-Modules 47

Chapter 2. The Spectral Sequence of a Bisimplicial Coalgebra 49
  1. Bisimplicial Θ-Coalgebras 50
  2. Filtrations 53
  3. The Spectral Sequence 55
  4. Bisimplicial Sets with Group Action 65
  5. Application to the Serre Spectral Sequence 71
  6. Application to André-Quillen Cohomology 73

Chapter 3. Bialgebra Actions on the Cohomology of Algebras 75
  1. Left Action by a Bialgebra 75
  2. Left Action by an Algebra with Coproducts 84
  3. Right Action by a Hopf algebra 89

Chapter 4. Extensions of Hopf Algebras 95
  1. Convolutions and Conjugations 96
  2. Some Properties of Extensions 102
  3. Adjunction Isomorphism and Change-of-Rings Spectral Sequence 107
Chapter 5. Steenrod Operations in the Change-of-Rings Spectral Sequence 113
1. The Spectral Sequence with its Products and Steenrod Squares 113
2. Steenrod Operations on $\text{Ext}^*_\Omega (P, \text{Ext}_Q^*(Q, N))$ 115
3. Central Extensions 120
4. The Operations at the $E_2$-level 120
5. A Simple Example 124
6. Application to the Cohomology of the Steenrod Algebra 126
7. Application to Finite sub-Hopf Algebras of the Steenrod Algebra 127
8. Applications to the Cohomology of Groups 129

Chapter 6. The Eilenberg-Moore Spectral Sequence 131
1. Kan Fibrations and Twisted Cartesian Products 132
2. Bisimplicial Models for Fiber Bundles 133
3. Construction of the Spectral Sequence 137
4. Calculation of the $E_2$-Term 138

Chapter 7. Steenrod Operations in the Eilenberg-Moore Spectral Sequence 141
1. The Spectral Sequence with its Products and Steenrod Squares 141
2. Steenrod Operations on $\text{Ext}^*_{H, G}(H_*E, K_*F)$ 142
3. The Operations at the $E_2$-level 144
4. Applications 148

Bibliography 149

Index 153