Contents

Preface xi

Basic Notation 1

Introduction 5

Chapter 0. Basic Concepts 17

1. Classification of the spectrum 17
2. Classes of compact operators 20
3. The resolvent equation. Conditions for self-adjointness 23
4. Wave operators (WO) 26
5. The smooth method 29
6. The stationary scheme 33
7. The scattering operator and the scattering matrix (SM) 38
8. The trace class method 42
9. The spectral shift function (SSF) and the perturbation determinant (PD) 45
10. Differential operators 52
11. Function spaces and embedding theorems 56
12. Pseudodifferential operators 58
13. Miscellaneous analytic facts 67

Chapter 1. Smooth Theory. The Schrödinger Operator 71

1. Trace theorems 71
2. The free Hamiltonian 75
3. The Schrödinger operator 79
4. Existence of wave operators 82
5. Wave operators for long-range potentials 86
6. Completeness of wave operators 93
7. The limiting absorption principle (LAP) 95
8. The scattering matrix 96
9. Absence of the singular continuous spectrum 98
10. General differential operators of second order 101
11. The perturbed polyharmonic operator 103
12. The Pauli and Dirac operators 104


1. Spectral analysis of differential operators with constant coefficients 109
2. Scalar differential operators 116
3. Nonelliptic differential operators 118
4. Matrix differential operators 122
5. Scattering problems for perturbations of a medium 124
6. Strongly propagative systems. Maxwell’s equations 128

Chapter 3. Scattering for Perturbations of Trace Class Type 133
1. Conditions on an integral operator to be trace class 133
2. Perturbations of differential operators with constant coefficients 136
3. The Schrödinger operator 139
4. The perturbed polyharmonic operator 145
5. General differential operators of second order 147
6. Scattering problems for perturbations of a medium 154
7. Wave equation 157
8. The scattering matrix and the spectral shift function 159

Chapter 4. Scattering on the Half-line 161
1. Jost solutions. Volterra equations 161
2. Generalized Fourier transform and WO 170
3. Low-energy asymptotics 178
4. High-energy asymptotics 188
5. The SSF for the radial Schrödinger operator 191
6. Trace identities 198
7. Perturbation by a boundary condition. Point interaction 203

Chapter 5. One-Dimensional Scattering 209
1. A direct approach 209
2. Low- and high-energy asymptotics 216
3. The SSF and trace identities 221
4. Potentials with different limits at “+” and “−” infinities 223

Chapter 6. The Limiting Absorption Principle (LAP), the Radiation Conditions and the Expansion Theorem 231
1. Absence of positive eigenvalues and radiation conditions 231
2. Boundary values of the resolvent 233
3. A sharp form of the limiting absorption principle 235
4. Nonhomogeneous Schrödinger equation 239
5. Homogeneous Schrödinger equation 241
6. Expansion theorem 245
7. The wave function. The scattering amplitude 251
8. A generalized Fourier integral 255
9. The Mourre method 259

Chapter 7. High- and Low-Energy Asymptotics 267
1. High-energy and uniform resolvent estimates 267
2. Asymptotic expansion of the Green function for large values of the spectral parameter 275
3. Small time asymptotics of the heat kernel 280
4. Low-energy behavior of the resolvent 285
5. Low-energy behavior of the resolvent. Slowly decreasing potentials 291

Chapter 8. The Scattering Matrix (SM) and the Scattering Cross Section 297
1. Basic properties of the SM 297
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>The spectrum of the SM. The modified SM</td>
<td>302</td>
</tr>
<tr>
<td>3</td>
<td>The scattering cross section</td>
<td>306</td>
</tr>
<tr>
<td>4</td>
<td>High-energy asymptotics of the SM. The ray expansion</td>
<td>311</td>
</tr>
<tr>
<td>5</td>
<td>The eikonal approximation</td>
<td>319</td>
</tr>
<tr>
<td>6</td>
<td>The averaged scattering cross section. Singular potentials</td>
<td>328</td>
</tr>
<tr>
<td>7</td>
<td>The semiclassical limit</td>
<td>337</td>
</tr>
<tr>
<td></td>
<td>Chapter 9. The Spectral Shift Function and Trace Formulas</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>The regularized PD and SSF for the multidimensional Schrödinger</td>
<td>341</td>
</tr>
<tr>
<td></td>
<td>operator</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>High-energy asymptotics of the SSF</td>
<td>353</td>
</tr>
<tr>
<td>3</td>
<td>Trace identities for the multidimensional Schrödinger operator</td>
<td>365</td>
</tr>
<tr>
<td></td>
<td>Chapter 10. The Schrödinger Operator with a Long-Range Potential</td>
<td>369</td>
</tr>
<tr>
<td>1</td>
<td>Propagation estimates</td>
<td>369</td>
</tr>
<tr>
<td>2</td>
<td>Long-range scattering</td>
<td>374</td>
</tr>
<tr>
<td>3</td>
<td>The eikonal and transport equations</td>
<td>380</td>
</tr>
<tr>
<td>4</td>
<td>Scattering matrix for long-range potentials</td>
<td>384</td>
</tr>
<tr>
<td></td>
<td>Chapter 11. The LAP and Radiation Estimates Revisited</td>
<td>399</td>
</tr>
<tr>
<td>1</td>
<td>The efficient form of the LAP</td>
<td>399</td>
</tr>
<tr>
<td>2</td>
<td>Absence of positive eigenvalues and uniqueness theorem</td>
<td>403</td>
</tr>
<tr>
<td>3</td>
<td>Nonhomogeneous Schrödinger equation with a long-range potential</td>
<td>408</td>
</tr>
<tr>
<td></td>
<td>Review of the Literature</td>
<td>415</td>
</tr>
<tr>
<td></td>
<td>Bibliography</td>
<td>429</td>
</tr>
<tr>
<td></td>
<td>Index</td>
<td>441</td>
</tr>
</tbody>
</table>