Introduction

In this book, our goal is to introduce the main notions, structures, and techniques used in quantum graph studies, as well as to provide a brief survey of more special topics and applications. This task has shaped the book as follows: we present in detail the basic constructions and frequently used technical results in Chapters 1 and 2, devoted to quantum graph operators, and Chapters 3 – 5, which address various issues of the spectral theory of quantum graphs. The remaining two chapters are of review nature and thus less detailed; in most cases the reader will be directed to the cited literature for precise formulations and proofs. Using graphs as models for quantum chaos is considered in Chapter 6. Chapter 7 provides a brief survey of various generalizations and applications. The reader will notice that the area is developing very fast; had we tried to be more specific in this chapter, it would be outdated by the time of publications anyway.

Our intent was to make the book accessible to graduate and advanced undergraduate students in mathematics, physics, and engineering.

Since a variety of techniques are used, for the benefit of the reader we introduce the main notions and relevant results in graph theory, functional analysis, and operator theory in a series of Appendices.

In order to make reading smoother, we normally do not include references in the main text of the chapters, collecting them, as well as additional comments, in the specially devoted last section of each chapter. We also have not tried to make the considerations too general. For instance, we mostly treat the second derivative operators on quantum graphs, while considerations could be easily extended to the more general Schrödinger operators. When we do mention more general operators, we do not look for the most general conditions on the coefficients (potentials), settling for some reasonable conditions that make the techniques work.