Contents

Preface vii

Introduction ix

Chapter 1. Heuristics of noise induced transitions 1
  1.1. Energy balance models of climate dynamics 1
  1.2. Heuristics of our mathematical approach 6
  1.3. Markov chains for the effective dynamics and the physical paradigm of spectral power amplification 14
  1.4. Diffusions with continuously varying potentials 18
  1.5. Stochastic resonance in models from electronics to biology 21

Chapter 2. Transitions for time homogeneous dynamical systems with small noise 27
  2.1. Brownian motion via Fourier series 28
  2.2. The large deviation principle 37
  2.3. Large deviations for Brownian motion 44
  2.4. The Freidlin–Wentzell theory 50
  2.5. Diffusion exit from a domain 59

Chapter 3. Semiclassical theory of stochastic resonance in dimension 1 69
  3.1. Freidlin’s quasi-deterministic motion 69
  3.2. The reduced dynamics: stochastic resonance in two-state Markov chains 78
  3.3. Spectral analysis of the infinitesimal generator of small noise diffusion 91
  3.4. Semiclassical approach to stochastic resonance 114

Chapter 4. Large deviations and transitions between meta-stable states of dynamical systems with small noise and weak inhomogeneity 133
  4.1. Large deviations for diffusions with weakly inhomogeneous coefficients 134
  4.2. A new measure of periodic tuning induced by Markov chains 144
  4.3. Exit and entrance times of domains of attraction 154
  4.4. The full dynamics: stochastic resonance in diffusions 169

Appendix A. Supplementary tools 177

Appendix B. Laplace’s method 179

Bibliography 183

Index 189