Preface

Philosophy

One of the most important questions we ask ourselves as teachers is "what do we want our students to remember about this course ten years from now?"

Our answer is sobering. From a ten year perspective most thoughts about the syllabus—"what should be covered"—seem irrelevant. What matters more is our wish to change the way our students' minds work—the way they approach a problem, or, more generally, the way they approach the world. Most people "skip the numbers" in newspapers, magazines, on the web and (more importantly) in financial information. We hope that in ten years our students will follow the news, confident in their ability to make sense of the numbers they find there.

To help them, we built this book around problems suggested by the news of the day as we were writing. We also consider issues that are common—and important—such as student loans. Common sense guides the analysis; we introduce new mathematics only when it's really needed. In particular, you'll find here very few problems invented just to teach particular mathematical techniques.

This preface is meant for students. There's also an instructor's manual that offers more detail about how we carry out our intentions. Students are free to read that too.

Organization

Most quantitative reasoning texts are arranged by topic—the table of contents reads like a list of mathematics to be mastered. Since the mathematics is only a part of what we hope to teach, we've chosen another strategy. Each chapter starts with a real story that can be best understood with careful reading and a little mathematics. The stories involve (among other things):

- Back of an envelope estimation.
- Discounts, inflation and compound interest.
- Income distribution in the United States.
- Reading an electricity bill.
- The graduated income tax.
- Reading a credit card bill.
- Paying off a mortgage or a student loan.

- Lotteries, gambling, insurance and the house advantage.
- False positives and the prosecutor's fallacy.

The best tool for understanding is common sense. We start there. When more is called for we practice "just in time" mathematics. The mathematics you need to understand a question appears when we ask the question. We don't ask you to learn something now because you'll need it later. You always see the mathematics in actual use.

This book differs from many at its level because we focus on how to *consume* numbers more than on how to *produce* them.

Paying attention to the numbers

We hope that when you've finished this course you will routinely look critically at the numbers you encounter every day. Questions like these should occur to you naturally:

- What do the numbers really mean?
- What makes them interesting (or not)?
- Are they consistent? Distorted?
- Do I believe them? Where do they come from?
- How might I check them?
- What conclusions can I draw from them?

To help you answer these kinds of questions we will think about:

- Relative and absolute change.
- Percentages.
- Units: all interesting numbers are numbers *of something*.
- Estimation skills. Counting zeroes: million, billions, trillions and beyond.
- Significant digits, orders of magnitude, quick and dirty mental arithmetic.
- Using a spreadsheet to ask "what-if" questions.
- Using a spreadsheet to display data.
- Models: when simple mathematics can clarify how data may be related.
- Probability and randomness.

Common sense and common knowledge

You can understand the numbers in a paragraph from the newspaper only if you understand something about the subject it addresses. Many of the discussions in the text and the exercises provide opportunities to explore—to learn things you might not know about economics or history or psychology or sociology or science or literature. When words or concepts are unfamiliar, or you're unsure of their meanings, look them up. Explore the ideas before you focus on the numbers.

The exercises

> In guessing a conundrum, or in catching a flea, we do not expect the breathless victor to give us afterwards, in cold blood, a history of the mental or muscular efforts by which he achieved success; but a mathematical calculation is another thing.
>
> Lewis Carroll
> *A Tangled Tale*
> Answers to Knot 4[R1]

Many textbooks give you a head start on the problems because they occur at the end of the section in which the relevant mathematics is taught. You need only look back a few pages to find a sample problem like the one you're working on. Often all you have to do is change the numbers.

There are few like that here. Most of ours call for a more extended solution—at least several sentences, sometimes several paragraphs. You can't simply calculate and then circle the answer on the page. On exams and on homework assignments we frequently remind you of what we expect with boilerplate like this:

> Be sure to write complete sentences. Show how you reached the answer you did. Identify any sources you used. When you refer to a website you should indicate why you think it is a *reliable* source—there's lots posted online that's just plain wrong.

Some exercises have hints at the back of the book. Try not to look at them until you've thought about the problem yourself for a while. If you invent or solve a problem and are particularly pleased with what you've done, send it to us and we'll consider incorporating it in a later edition of the book, with credit to you, of course. If you find an error, please let us know.

When you think you've finished an exercise, read your answer carefully just to see that it makes sense. If you estimate an average lawyer's annual income as $10,000 you have probably made a mistake somewhere. It's better to write "I know this is wrong, but I can't figure out why—please help me" than to submit an answer you know is wrong, hoping no one will notice.

We've annotated some of the exercises. Here's what the tags mean:

- [S]: The solution manual contains an answer.
- [U]: This exercise is untested. We think it might be a good one, but haven't yet tried it out in a class.
- [C]: This exercise is complex, or difficult, or ambiguous. Many problems in the real world are like that, so this book has a few too.
- [W]: This is a worthy exercise. It's particularly instructive, perhaps worth taking up in class.
- [R]: A routine exercise.
- [A]: An exercise with artificial numbers. Sometimes problems like this are good for emphasizing particular points, but we try to avoid them when we can.
- [N]: The idea for an exercise, with no details yet.
- [Goal *x*.*y*]: Contributes to mastery of Goal *y* of Chapter *x*.
- [Section *x*.*y*]: Depends on or adds to material in Section *y* of Chapter *x*.

The world is a messy place

When you're reading the newspaper or come across a web page or see an ad on television you're not told which chapter of the book will help you understand the numbers there. You're on your own.

When we ask open ended questions like those triggered by the news of the day our students are often uncomfortable. Here are comments expressing that discomfort, from two students, part way through the course:

- I still don't understand sometimes how we are given questions that are almost meant to confuse the reader.
- The only improvements I would make would be that we have more structured problems for homework, occasionally they can be broad.

But at the end of the semester students wrote anonymously in answer to the question "What are the strong points of this course?"

- In Quantitative Reasoning we are learning how to look into numbers instead of just looking at them.
- Use math in everyday settings instead of thinking when will I ever use this.
- It covers a real-world perspective of math.
- This course taught material that will be extremely helpful in the future.
- It teaches math that can be used every day, and skills in Excel that are useful and that I will definitely be using down the line.
- This course is very useful for me in the outside world and I feel that I will benefit from the education I received from this class and I will be able to apply my new knowledge to situations outside of the classroom.
- This course has taught me a lot about obtaining information, and using it in ways that I had not before.
- Very applicable subject matter for other areas of academics and professions. The course was something of a blend of refresher mathematics, and a new ways to apply them to everyday life.
- The math was more interesting, relevant. Good examples, news articles employed.
- I think that hardest part about this class was thinking. When you usually enter a math class the only thinking that you have to do is remember equations but in this math class I had to do research and find things on my own to help me out to answer a question.
- For the love of all that is good, why is an English major/poet/musician forced to take math all these years? I am not well-rounded or more comfortable with math, it has just drawn out my college career, costing me time and money that I don't have. I will never use math in my life, the types that I will employ I learned in elementary school. This was the best math class I have ever taken though.

Real and up to date

Our philosophy demands that the examples and exercises in *Common Sense Mathematics* pose real questions of genuine interest. Therefore they usually come from the news of the day at the

time we wrote them. You don't have to go to the original sources to answer the questions, but if you're curious you can. You will find the bibliographic details in the References section at the end of the book.

One problem with our philosophy is that the text is out of date as soon as it's printed. Our remedy is to rewrite the course and add to the exercises on the fly each time we teach it. We hope other instructors will do that too. That way nothing is ever stale, which is good. What's less good is that the discussions here may not correspond to what actually happens in the course you are taking.

Common Sense Mathematics on the web

The home page for this text is `commonsensemathematics.net`. There you will find the spreadsheets we refer to, a link to a teaching blog for the years 2008-2014, errata we find (or you tell us about) and other information teachers and students of quantitative reasoning might find useful.

Technology

We wrote this text to help students understand questions where quantitative reasoning plays a part. To that end, we take advantage of any tools that will reduce drudgery and prevent careless errors.

For most applications, an ordinary four function calculator will do—and these are ubiquitous. You probably have one on your cell phone. When more advanced arithmetic is called for you can use a spreadsheet or the calculator on your computer or the internet.

You'll find references to websites in the exercises and elsewhere throughout the book. The references were accurate when we wrote the book, but we know that the web changes. If a link doesn't work, don't give up. Most likely it's moved somewhere else. A broken link isn't an excuse to skip a homework problem. Instead, be resourceful. Look around on the web or email us or your instructor.

We think an educated citizen these days should be able to refer to the internet wisely and effectively and be comfortable using a spreadsheet. In *Common Sense Mathematics* we use Excel, not because we are particularly fond of Microsoft, but because it is the most common spreadsheet in use today. But almost all our spreadsheets can be recreated in any spreadsheet program now or (we imagine) in the near future.

A spreadsheet program is good for data analysis, for asking "what if" questions and for drawing graphs. It also helps make mathematical abstractions like "function" real, rather than formal. We introduce Excel in Chapters 6 and 7. From then on we ask you to create spreadsheets, and to use simple ones we've built for you—more complex than ones you could write, but not too hard to read and understand as well as use.

We regularly refer to searching with Google, because it is the most commonly used search engine. But any other should do; use your favorite.

One search feature turns out to be particularly useful. Both Google and Bing will do arithmetic for you when you type a numerical calculation in the search field.

Old vs. new

If you know one way to do a problem should you learn another? That depends. ("That depends" is the answer to most interesting questions. If the question calls for a straightforward "yes" or "no" or just a number or something you can discover in one step with a web search the question is probably not very interesting.)

If you rarely encounter similar problems it's not worth the effort needed to understand and remember a new way to do them. But if you expect to see many, then it may pay to learn that new method.

For example, if you plan to spend just a day or so in a foreign country, get a phrase book with the common words you'll need to communicate. But if you plan to live there half the year, learn the language.

Here's a second example. When using a computer, there are many things you can do with either the mouse or the keyboard. The mouse is intuitive. You can see just what's happening, and there's nothing to remember. Just pull down the menu and click. But the keyboard is faster. So if you're going to do something just once or twice, use the mouse, but if you're going to do it a lot, learn the keyboard shortcut. In particular, in computer applications these days you often copy text from one place to paste it in another, whether that's from a web page, or in your word processor or spreadsheet. You can do that from the edit menu, or you can use the keyboard shortcuts `control-C` and `control-V`. Learn the shortcuts!

We have tried in this book to teach you new ways to do things when we think those new ways will serve you well in the future. We've resisted the temptation when those new ways are just clever tricks mathematicians are fond of that don't really help you in the long run.

Truth and beauty

We've worked to limit the mathematics we cover to just what you need, along with common sense and common knowledge, to help you deal with the quantitative parts of a complex world. But there is another important reason to study mathematics.

You read not only because it's useful, but because reading can give you access to poetry. You cook not only because you must eat to live, but because there can be pleasure in preparing tasty meals and sitting down in good company to enjoy them. We became mathematicians not only because mathematics is useful, but because (for us and some other people like us) it's beautiful, too.

This passage from Henry Wadsworth Longfellow's 1849 novel *Kavanagh* captures both the truth and the beauty of mathematics (as we hope we have).

> "For my part," [says Mary Churchill] "I do not see how you can make mathematics poetical. There is no poetry in them."
>
> "Ah, that is a very great mistake! There is something divine in the science of numbers. Like God, it holds the sea in the hollow of its hand. It measures the earth; it weighs the stars; it illumines the universe; it is law, it is order, it is beauty. And yet we imagine—that is, most of us—that its highest end and culminating point is book-keeping by double entry. It is our way of teaching it that makes it so prosaic." [R2]

Contact us

We welcome questions, feedback, suggested problems (and solutions) and notes about errors. You may contact us by email at `eb@cs.umb.edu` (Ethan Bolker) or `mmast@fordham.edu` (Maura Mast).

Acknowledgements

We owe much to many for help with *Common Sense Mathematics*.

Many years ago Linda Kime shepherded the first quantitative reasoning requirement at UMass Boston. In 2007 then Mathematics Chair Dennis Wortman allowed us to coteach Math 114 in hopes of reinventing the course. In early years Mark Pawlak pilot tested early versions of this text. He was the first to believe that we were onto a good thing—soon he was scouring the newspaper for examples to use in class, on exams, and in the exercises. His input, drawn from his long involvement in quantitative reasoning and his deep engagement with student learning, has made this a better book. When *Common Sense Mathematics* became the official textbook for quantitative reasoning at UMass Boston Mark was the course administrator, trained tutors, developed new approaches to assessing student learning and recruited instructors: George Collison, Karen Crounse, Dennis DeBay, Monique Fuguet, Matt Lehman, Nancy Levy, John Lutts, Robert Rosenfeld, Jeremiah Russell, Mette Schwartz, Joseph Sheppeck, Mitchell Silver, Karen Terrell, Charles Wibiralske and Michael Theodore Williams.

We benefited from feedback from colleagues at other schools who asked to use the text: Margot Black (Lewis & Clark), Samuel Cook (Wheelock College), Grace Coulombe (Bates College), Mike Cullinane (Keene State College), Timothy Delworth (Purdue University), Richard Eells (Roxbury Community College), Marc Egeth (Pennsylvania Academy of the Fine Arts), Ken Gauvreau (Keene State College), Krisan Geary (Saint Michael's College), David Kung (St. Mary's College of Maryland), Donna LaLonde (Washburn University), Carl Lee (University of Kentucky), Alex Meadows (St. Mary's College of Maryland), Wesley Rich (Saginaw Chippewa Tribal College), Rachel Roe-Dale (Skidmore College), Rob Root (Lafayette College), Barbara Savage (Roxbury Community College), Q. Charles Su (Illinois State University), Joseph Witkowski (Keene State College) and several anonymous reviewers.

Students caught typos, suggested rewordings and provided answers to exercises. We promised to credit them here: Courtney Allen, Matt Anthony, Vladimir Altenor, Theresa Aluise, Selene Bataille, Kelsey Bodor, Quonedell Brown, Katerina Budrys, Candace Carroll, Jillian Christensen, Katie Corey, Molly Cusano, Sam Daitsman, Hella Dijsselbloem-Gron, Michelle DiMenna, Shirley Elliot, Lea Ferone, Solomon Fine, Murray Gudesblat, Frances Harangozo, Irene Hartford, Katilyn Healey, Anna Hodges, Anthony Holt, Laura Keegan, Jennifer Kunze, Kevin Lockwood, Jacob Looney, Ashley McClintock, Edward McConaghy, Nicole McKenna, Amanda Miner, Antonio de las Morenas, Daniel Murano, Hannah Myers, Matt Nickerson, Rodrigo Nunez, Gabby Phillips, Jaqueline Ramirez, Hailey Rector, Taylor Spencer, Jaran Stallbaum, Melinda Stein, Willow Smith, Nick Sullivan, Robert Tagliani, Julia Tran, Marcus Zotter, ... and many others.

Cong Liu worked on the index. Monica Gonzalez and Alissa Pellegrino tested the links to the web. Paul Mason ferreted out the newspaper headlines for the cover.

The National Science Foundation provided support from grant DUE-0942186. Any opinions, findings and conclusions or recommendations expressed in this material are those of the contributors and do not necessarily reflect the views of the National Science Foundation. We hope they approve of what we've done with their generosity.

The Boston Globe graciously gave us blanket permission to reproduce here the quotes we found in our morning paper and brought to class.

Wizards at `tex.stackexchange.com` were always quick to answer TEXnical questions.

Carol Baxter, Stephen Kennedy, Beverly Ruedi and Stanley Seltzer at the Mathematical Association of America were enthusiastic about our book and brought wisdom and competence to design and production.

Ethan: My wife Joan's ongoing contribution began 56 years ago when she asked me how I'd feel if I went to medical school and did no more mathematics. It continued with constant support of all kinds—most of the details would be inappropriate here. I will say that I recommend living with a writing coach to hone writing strategies. I've talked for years with my professor children about mathematics and teaching—Jess and Ben make cameo appearances in the text. I dedicate *Common Sense Mathematics* to the next generation: Solomon Bixby and Eleanor Bolker.

Maura: I owe a greater debt than I could ever express to my husband Jack Reynolds. We met in Boston over 21 years ago (thanks in part to a National Science Foundation grant). Living in Iowa three years later, we saw that UMass Boston wanted to hire a mathematician to work on quantitative reasoning. With Jack's support, I left a tenured position to accept that challenge. I couldn't have done that and my other work at UMass as well as I did without his faith in me and his support. There's more. Jack has been my conscience as well as my partner. He fundamentally believes that each person can make a difference in the world. Because of that, I now see my work in quantitative reasoning, and mathematics, as a way to change the world. I thank my children Brendan, Maeve and Nuala Reynolds for their patience and support, especially for the times when I turned their questions into homework problems, as in the tooth fairy exercise. I dedicate *Common Sense Mathematics* to the memory of my parents, Cecil and Mary Mast, who set high ideals grounded in reality. I miss them dearly.

Newton, MA and The Bronx, NY
October 2015