Mathematics has been a major force shaping human lives for several millennia. Some of humanity’s deepest thoughts are expressed in the language of mathematics, and many of its most powerful tools could not have been devised without it. At various stages in history mathematics has been highly regarded and at others largely neglected. Some people find it a source of delight, some treat it as a mere tool, some find it cold and lifeless. The study of the history of mathematics offers us a way to understand the many roles that mathematics has played, and continues to play, in the lives of societies, cultures, and individuals. We can use it to shed light on the nature of mathematics itself, and perhaps on some of the activities that make us distinctively human.

We first see mathematics at work in the creation of the great civilisations of the ancient Near East: Egypt and Mesopotamia. Then we turn to the later culture of ancient Greece, and see the emphasis placed on abstraction and proof. The classic text from this period is Euclid’s *Elements*, often claimed to be one of the most published books ever written, and taught in schools in one form or another as late as the 1960s. This is a remarkable life for a book written well over 2000 years ago. What does it tell us about the work itself, and the people who used it, that it lasted so long? What are the implications of such longevity?

Later we consider some of the roles that mathematics played in the history of China and India, and in the early centuries of the world of Islam. How did mathematicians in these societies see the nature and purpose of mathematics? What did they contribute to it?

We then turn to the reawakening of intellectual life in Europe, which brought with it a renewed interest in the lost arts of mathematics, driven variously by trade, a desire to catch up with the Islamic world, and also by astronomical, scientific, and ultimately abstract intellectual issues. We conclude by looking at the work of Fermat and Descartes in the 17th century.

The topics covered here are all mainstream, important topics in the history of mathematics, but even in a book of this size some difficult choices had to be made. We have sought to include topics that carry a broad significance. So the chapter on Egyptian and Mesopotamian mathematics not only covers some of the earliest evidence we have of mathematical activity, it points firmly towards the idea that mathematics, however it is defined, is not a Greek invention. The same message is, of course, conveyed in the chapters on mathematics in India, China, and the Islamic world. Scholarship on mathematics in Egypt
and Mesopotamia was largely the creation of Otto Neugebauer in the middle of the 20th century, and it has been enjoying a revival recently; comparably good work on India, China, and the Islamic world is mostly recent and growing. All these fields offer new perspectives on the nature of mathematics, its connection to proof, its uses in the societies that have sustained it, and, at least at times, the people who created it.

None of this diminishes the achievements of Greek mathematicians, either in their own day or in the centuries of the revival in the West. We have concentrated on their geometry, because that was not only a highlight but, through its emphasis on certain kinds of proof, was to have a decisive effect on the image of mathematics in later times. But we have also looked at the connections to philosophy and astronomy, and at the rather different accomplishments of Greek mathematicians in the Hellenistic period. We have also followed these stories into the Islamic world, and we look at the implications of those mathematicians for later Western work on algebra in particular. We end this account with two chapters: one on astronomy and one on the synthesis of algebra and geometry particularly associated with Descartes.

Studying the history of mathematics is by no means the same as studying mathematics itself, although some familiarity with mathematics is advisable. The questions that this book addresses are questions about the history of mathematics, not mathematical questions with a historical flavour (exciting though that can be, too). We are interested in understanding who did the mathematics, and why? Were they teachers — and if so, who were their students and why were they there? Was there a cultural or philosophical dimension to their mathematical work? What does it mean to discover something in mathematics? How was mathematical knowledge disseminated? Surprisingly rich answers to questions such as these can be obtained without one having to master the accompanying mathematics. What was done is interesting, but why it was done is interesting too. What was the context that made the work important in its day? Why is it still of interest to study today? These are the central questions that flow through this book.

To answer these questions, historians of mathematics rely on what we might call the ‘facts’, which can be drawn from many sources: documents, written texts, and also various artefacts. The historian’s task is to make these mute objects speak again — but inevitably they do so with the historian’s voice. This is ultimately because the big questions raised here do not have simple answers, and so studying the history of mathematics involves a certain amount of disagreement. Historians produce arguments, based on selections of the evidence; their conclusions are not so much facts as opinions. We can ask that their opinions are well-argued and well-informed, but opinions they remain, and other historians, perhaps bringing forth new facts, can disagree. This gives the history of mathematics a necessarily provisional character, but it opens the way to new and important work in the subject, and you will find much recent historical work reflected in these pages.

Because our intention is to bring original sources to life whenever appropriate, our approach raises the question of the prerequisites for reading this book. These are both more and less than a casual inspection might suggest. More, because the material on geometry has become increasingly remote. Less, because our interest is in the history of the mathematics we introduce, and it is often possible to see and appreciate the importance of a source without entirely grasping the mathematics it contains. We can often see what a text is about, how it differs from other comparable accounts, and why it was written even when the topic is technical and the exposition difficult to follow. And, depending on the historical question being investigated, that is often enough.

We do not assume knowledge of any specific piece of mathematics, but we assume some familiarity with the subject and a willingness to grapple with the details. The sources,
and our commentaries on them, should be read pen in hand; drawing a diagram for yourself is the best way of seeing what is in it. We have also provided suggested exercises, many of which contain primary sources, and which — unlike the exercises in most textbooks on the subject — are firmly historical, rather than largely mathematical. They will be found at the end of the book, along with suggestions for how they can be approached.

This book is aimed at the general mathematically inclined reader. We hope that it will provide a rich introduction not only to the history of mathematics, but to mathematics itself. The prerequisites gradually involve more mathematics, but we believe that each section of this book can be read as an introduction to the mathematics involved, as well as providing an absorbing account of history it describes.

No-one should have left school with too little mathematics to follow this book. But for those who were cut off from mathematics because they could not appreciate why it was done, and who could not connect to its excitements, we hope that this historical account offers a rewarding way in.