## Index

absorbing Markov chain, 316, 320
absorbing state, 287, 316, 320
algebraic multiplicity, 529
alternating determinants, 238
angle between two vectors, 10, 73
angle of rotation, 223, 224, 225
angle-sum identities, 234
attractor, 309
augmented matrices, 114
axis of rotation, 223

basic rules
of cross product, 79, 500
of arithmetic with points, 14, 16, 157, 343, 377
of determinants, 238, 458, 471
of dot product, 56, 65
of generalized cross product, 502
of matrix algebra, 156, 215, 343, 377

cbasis for a vector space, 80, 369, 394, 411
bijective, 411
block diagonal matrix, 197
block triangular matrix, 197
Blow-Up Theorem, 396

Cauchy-Schwarz Inequality, 72, 74, 83
Cayley-Hamilton Theorem, 534
center of rotation, 223
change of basis matrix, 424
Change of Basis Theorem, 424
Change of Representation Theorem, 425
characteristic equation, 513
characteristic polynomial, 513, 514, 529, 534
characteristic vector, see eigenvector
closed, 342, 343, 353, 355
coefficient matrix, 114
column rank, 400, 403
column space, 358, 361
component, 65, 71
constructive proof, 396
coordinate, 8, 12
coordinate equation, 4, 88
of a plane, 31, 91
coordinate vector, 372, 375, 394, 415, 422
Cramer’s Rule, 496, 499
cross product, 54, 78, 236, 454, 466, 494, 500
generalized, 501
determinant, 78, 174, 237, 451, 454, 466, 477
product rule for, 243, 483
diagonal matrix, 160, 473, 525
diagonalizable matrix, 525, 529, 532
dimension, 376, 400, 402, 403, 405
direction vector, 90, 92, 94
distance between two points, 38, 39, 51
dot product, 52, 65, 71, 113, 166, 171, 454, 456
dynamical system, 535
echelon form, 119, 132, 137, 138, 190, 218, 251, 252, 311, 394, 401, 479, 482
eigenspace, 518, 530
eigenvalue, 435, 510
eigenvector, 197, 262, 331, 435, 510, 510, 526, 527
elementary row matrix, 218, 479
elementary row operations, 116, 132, 400, 477–479
equal matrices, 155
points, 12
equivalence of matrices, 114
of systems, 114
of vectors, 20, 22
Euclidean space, 12
even-odd flip test, 468
Extension by Linearity Theorem, 411
extension program, 22, 38, 53, 60, 70, 90, 93, 94, 96, 155, 239, 248, 454, 491, 502
faithful representation, 374, 411
Fatter Than Tall Theorem, 258, 369
Fibonacci sequence, 351
finite dimensional, 369
fixed lines, 508
fixed vector, 221, 263
Index

flip, see transposition
function, 178, 213, 225
function composition, 249, 260, 261

Gaussian elimination, 117, 189
generating system, 363, 393, 404
geometric multiplicity, 530
Google, 329
graph, 8

head minus tail test, 21
head of a vector, 19
homogeneous system, 122, 136, 185, 258
hyperplane, 94
equation of, 91, 95

identity mapping, 419, 422
identity matrix, 133, 138, 179, 190, 302, 422, 458
image, 250
of a matrix, 247, 253, 257, 260
of a vector, 247
infinite dimensional, 369
initial point, 19, 94, 95
invariant subspace, 517
invariant vector space, 517
inverse, 355, 433
invertible matrix, 179, 186, 188, 258, 433
kernel, 136, 185, 188, 252, 256, 257, 302, 401–403, 405, 429

Lagrange Identity, 79, 83
lattice point, 61
Law of Cosines, 69, 74
length, 36, 51, 59, 69, 71
line, 86, 90, 136, 154, 291

linear combination, 17, 28, 31, 96, 125, 127, 130, 177, 256, 258, 260, 300, 359, 361, 362, 395, 411, 512, 528
linear equation, see coordinate equation
linear map, 213, 225, 239, 243, 248, 257, 403, 409, 411
linear span, 139, 359, 400
linear transformation, 213, 215, 223, 225–227, 409, 424, 510
linearly dependent, 131, 258, 369, 462
linearly independent, 131, 139, 191, 369, 375, 394, 395, 404, 405, 489, 496, 512, 526, 527

M-cyclic subspace generated by Q, 523
magnitude, see length
Markov chain, 284, 306, 316, 536
Markov chains, 524
mathematical induction, 473
matrix, 114, 151, 154, 291
entry, 154
equality of, 155
inverse, 179, 217
matrix for a linear map with respect to two bases, 416

matrix multiplication, 167, 176, 229, 249, 261, 298–300
properties, 178
Matrix Power Formula, 301
matrix product, 167, 482
maximal linearly independent set, 396
midpoint, 34
minors, 454
multiplication
of a matrix by a scalar, 156
of a point by a scalar, 13, 14, 25, 26
of a vector by a scalar, 24, 27, 37, 54, 64, 508
of two matrices, 167, 176
mutually orthogonal, 53

n-dimensional Euclidean space, 12
node, 290
nonsingular matrix, 179
normal, 94, 95
one-to-one function, 258
ordered n-tuple, 12
orthogonal, 53
matrix, 221, 228
vectors, 51, 53, 54, 59, 64, 76, 136, 454, 466, 492
PageRank, 330
parallel, 26, 87
parallelepiped, 472, 489, 492
volume spanned by n vectors, 502, 504
volume spanned by three vectors, 491, 502
parallelogram, 13, 30, 489
area spanned by two vectors, 82, 236, 237, 240, 243, 491, 502
Parallellogram Rule, 13, 24, 30
parallelootope, 491
parameter, 90, 94
parametric equation, see vector equation
Pick-Apart Theorem, 178, 226, 240, 243, 300, 308, 317, 404, 412, 531
plane, 93, 489
point, 12, 19
equality of, 12
point-tester, 9, 31, 86
polynomial, 345, 354
preimage, see pullback
probability, 283, 284, 290, 292
probability vector, 297, 305, 537
projection, 65, 69, 71, 259, 410, 490
pullback, 250, 257, 259
Pythagorean Theorem, 35, 51, 59, 60
quadratic form, 532

random process, 282, 535
rank, 405, 405
real vector space, 343
representation, 374
rotation, 223, 410
rotation matrix, 228, 426, 515, 532
row rank, 400
row space, 361, 400
row-reduced echelon form, see echelon form

scalar, 13
scalar multiple
  of a vector, 343
  of a matrix, 156, 162
  of a point, 25, 225
  of a vector, 409
scalar triple product, 84
sign matrix, 453, 456, 468
similar matrices, 184, 433, 516
singular matrix, 179
skew, 134
slope, 20, 51
spanned, 29
square matrix, 139, 160, 527
standard basis vectors, 80, 308, 415
steady state, 301, 305, 318, 336
structure preserving, 373, 411
submatrix, 322
subspace, 354, 517
subtraction of points, 15, 21, 26, 38
sum of matrices, 156
sum of points, 12, 14, 26
system of equations, 116
system of equations, 17, 55, 114, 115, 118, 120, 127, 137, 185, 190, 256, 258, 492, 499
tail of a vector, 19
terminal point, 19
TFAE Theorem, 191, 371, 377, 405, 485, 495, 497
trace, 174, 438
transient state, 316
transition matrix, 305
transition graph, 290
transition matrix, 291, 299, 316, 320, 330, 537
transition probability, 290
translation, 23, 188
transpose, 473
transpose of a matrix, 157, 161
transposition, 449, 468
Triangle Inequality, 37, 73
triangular matrix, 161, 197, 474, 478
trivial solution, 130
unit vector, 37, 300
Vandermonde determinant, 476
vector, 19, 20, 297, 343, 359
class, 23
equivalence, 20, 22
head, 19
orthogonality, 51, 53, 76
tail, 19
vector equation, 88
  of a line, 27, 90
  of a plane, 31, 94

vector space, 343, 353, 409
well-defined, 458, 466
x-axis, 8, 431
y-axis, 8, 431
zero matrix, 156
zero vector, 23, 137, 185, 343, 355