Contents

Introduction vii

Chapter 0. Three basic definitions and three principal theorems 1

Part I. The beginning: Spaces and operators 7

Chapter 1. Preparing the stage 9
  1.1. Operators on normed spaces 9
  1.2. Operators on Hilbert spaces 11
  1.3. The diamond multiplication 14
  1.4. Bimodules 15
  1.5. Amplifications of linear spaces 16
  1.6. Amplifications of linear and bilinear operators 19
  1.7. Spatial tensor products of operator spaces 21
  1.8. Involutive algebras and $C^*$-algebras 24
  1.9. A technical lemma 30

Chapter 2. Abstract operator (= quantum) spaces 33
  2.1. Semi-normed bimodules 33
  2.2. Protoquantum and abstract operator (= quantum) spaces. General properties 36
  2.3. First examples. Concrete quantizations 38

Chapter 3. Completely bounded operators 47
  3.1. Principal definitions and counterexamples 47
  3.2. Conditions of automatic complete boundedness, and applications 51
  3.3. The repeated quantization 57
  3.4. The complete boundedness and spatial tensor products 59

Chapter 4. The completion of abstract operator spaces 63

Part II. Bilinear operators, tensor products and duality 67

Chapter 5. Strongly and weakly completely bounded bilinear operators 69
  5.1. General definitions and properties 69
  5.2. Examples and counterexamples 70

Chapter 6. New preparations: Classical tensor products 75
  6.1. Tensor products of normed spaces 75
  6.2. Tensor products of normed modules 78

Chapter 7. Quantum tensor products 81
7.0. The general universal property 81
7.1. The Haagerup tensor product 82
7.2. The operator-projective tensor product 90
7.3. The operator-injective tensor product 96
7.4. Column and row Hilbertian spaces as tensor factors 101
7.5. Functorial properties of quantum tensor products 106
7.6. Algebraic properties of quantum tensor multiplications 111

Chapter 8. Quantum duality 119
8.1. Quantization of spaces in duality 119
8.2. Quantum dual and quantum predual space 123
8.3. Examples 126
8.4. The self-dual Hilbertian space of Pisier 132
8.5. Duality and quantum tensor products 136
8.6. Quantization of spaces, set in vector duality 139
8.7. Quantization of the space of completely bounded operators 141
8.8. Quantum adjoint associativity 145

Part III. Principal theorems, revisited in earnest 151

Chapter 9. Extreme flatness and the Extension Theorem 153
9.0. New preparations: More about module tensor products 153
9.1. One-sided Ruan modules 156
9.2. Extreme flatness and extreme injectivity 158
9.3. Extreme flatness of certain modules 160
9.4. The Arveson–Wittstock Theorem 163

Chapter 10. Representation Theorem and its gifts 167
10.1. The Ruan Theorem 167
10.2. The fulfillment of earlier promises 171

Chapter 11. Decomposition Theorem 177
11.1. Complete positivity and the Stinespring Theorem 177
11.2. Complete positivity and complete boundedness: An interplay 180
11.3. Paulsen trick and the Decomposition Theorem 183

Chapter 12. Returning to the Haagerup tensor product 189
12.1. Alternative approach to the Haagerup tensor product 189
12.2. Decomposition of multilinear operators 193
12.3. Self-duality of the Haagerup tensor product 197

Chapter 13. Miscellany: More examples, facts and applications 201
13.1. CAR operator space 201
13.2. Further examples 210
13.3. Schur and Herz–Schur multipliers 219

Bibliography 231
Index 237